"Design and Fabrication of Experimental Set-up for Study of Vibrations in Academia"

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in
Mechanical Engineering

by

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Project Approval Sheet

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Dedicated to our parents

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ABSTRACT

Study of the subject of 'vibrations' is becoming increasingly important as most human activities involve vibration in one form or the other. In a world driven by prime movers which have an inherent imbalance, vibration related problems are encountered in everyday industrial and domestic use. Modelling a system for characterization of its vibration properties for several input conditions and initial configurations involves set of complex equations and hence, the theory of vibrations remains one of the most abstract field in the study of physics and engineering. Numerical simulations, animations and experimental setups for different vibration systems can thus be a very useful tool in making students understand the subject better. This report is based on a semi-portable and low-cost vibration demonstration equipment which covers two discrete systems and a continuous system in the classical vibrations textbooks. A simple pendulum and a double pendulum is accommodated in a single modular pendulum set-up. A string fixed at both ends under a specified tension is chose as a continuous degree of freedom system. The experimental setup for a single, two degree of freedom system and a continuous system is explained in detail. Derivations for the equations of motions in both polar and Cartesian coordinates for the pendula is covered. It is limited to Cartesian coordinates in case of the chosen continuous degree of freedom system. Study of motion governed by these equations is carried out with the help of a commonly available web camera and video processing software. Results obtained from the motion analysis are compared against the theoretical solution available for the system to establish accuracy of the vision based approach. This set-up is intended for use in classroom demonstrations and laboratories. The technique may be extended to other experimental studies which could be advantageous to underdeveloped and developing countries in setting up various laboratories; a process which otherwise would be highly cost and technology intensive. A multi-parameter condition monitoring setup is also proposed, with primary input being a vision based sensor.

The report is divided into seven chapters and is structured as follows, Chapter 1 discusses the historical background of the field of vibrations and the motivation, scope and objective of this study, Chapter 2 covers a brief overview of common vibration systems. We present the literature review on the subject of this report in Chapter 3. Chapters 4 and 5 cover the experimentation, theoretical derivations and comparison of the two in quite some detail. The idea of vision based condition monitoring system is presented in Chapter 6 followed by conclusions drawn and discussion about the future scope of this project in Chapter 7.

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LIST OF SYMBOLS

ℓ	Length of simple pendulum string
D^g	Distance of pixels in global metric coordinates (*mm*)
D^p	Local pixel distance (*pixel*)
κ	Global distance per unit pixel (*mm/pixel*)
g	Acceleration due to gravity.
Δt	Time interval between two successive frames
v	Velocity of oscillations
X_0	Peak amplitude of oscillations
x_d	Damped amplitude of oscillations
ω_n	Natural frequency of the oscillating system
ω_0	Initial frequency of oscillation
θ_1	Angle from vertical of bob 1 of double pendulum
θ_2	Angle from vertical of bob 2 of double pendulum
l_1	Length of upper string of bob 1 of double pendulum
l_2	Length of lower string of bob 1 of double pendulum
m_1	Mass of bob 1 or upper bob of double pendulum
m_2	Mass of bob 2 or lower bob of double pendulum
ξ	Damping coefficient
T	Time period of oscillations of pendulum

LIST OF ABBREVIATIONS

DOF	Degree of Freedom
EOM	Equation of Motion
VBVS	Vision Based Vibration Signature
USB	Universal Serial Bus
EPM	Experiment Performed Manually
EPI	Experiment Performed using Image processing
fps	Frames per second
ESPI	Electronic Speckle Pattern Interferometry
CCD	Charge Coupled Device
LVDT	Linear Variable Differential Transformer
LED	Light Emitting Diode
LDV	Laser Doppler Vibrometers
MR	Magnetorheological
FEM	Finite Element Method
AE	Acoustic Emission
FRF	Frequency Response Function
NDT	Nondestructive Testing
VAM	Vibroacoustic Modulation

CHAPTER 1: INTRODUCTION

1.1 HISTORICAL NOTE ON VIBRATION

People became interested in vibration when they created the first musical instruments, probably whistles or drums. Since then, both musicians and philosophers have sought out the rules and laws of sound production, used them in improving musical instruments, and passed them on from generation to generation. As long ago as 4000 B.C., music had become highly developed and was much appreciated by Chinese, Hindus, Japanese, and, perhaps, the Egyptians. These early peoples observed certain definite rules in connection with the art of music, although their knowledge did not reach the level of a science. Stringed musical instruments probably originated with the hunter s bow, a weapon favored by the armies of ancient Egypt. One of the most primitive stringed instruments, the nanga, resembled a harp with three or four strings, each yielding only one note. An example dating back to 1500 B.C. can be seen in the British Museum. The Museum also exhibits an 11-stringed harp with a gold-decorated, bull-headed sounding box, found at Ur in a royal tomb dating from about 2600 B.C. As early as 3000 B.C., stringed instruments such as harps were depicted on walls of Egyptian tombs. Our present system of music is based on ancient Greek civilization. The Greek philosopher and mathematician Pythagoras (582-507 B.C.) is considered to be the first person to investigate musical sounds on a scientific basis. Among other things, Pythagoras conducted experiments on a vibrating string by using a simple apparatus called a monochord. Pythagoras observed that if two like strings of different lengths are subject to the same tension, the shorter one emits a higher note; in addition, if the shorter string is half the length of the longer one, the shorter one will emit a note an octave above the other. Pythagoras left no written account of his work, but it has been described by others. Although the concept of pitch was developed by the time of Pythagoras, the relation between the pitch and the frequency was not understood until the time of Galileo in the sixteenth century.

Around 350 B.C., Aristotle wrote treatises on music and sound, making observations such as the voice is sweeter than the sound of instruments, and the sound of the flute is sweeter than that of the lyre. In 320 B.C., Aristoxenus, a pupil of Aristotle and a musician wrote a three-volume work entitled Elements of Harmony. These books are perhaps the oldest ones available on the subject of music written by the investigators themselves. In about 300 B.C., in a treatise called Introduction to Harmonics, Euclid, wrote briefly about music without any reference to the physical nature of sound. No further advances in scientific knowledge of sound were made by the Greeks. It appears that the Romans derived their knowledge of music completely from the Greeks, except that Vitruvius, a famous Roman architect, wrote in about 20 B.C. on the acoustic properties of theaters. His treatise, entitled De Architectura Libri Decem, was lost for many years, to be rediscovered only in the fifteenth century. There appears to have been no development in the theories of sound and vibration for nearly 16 centuries after the work of Vitruvius. China experienced many earthquakes in ancient times. Zhang Heng, who served as a historian and astronomer in the second century, perceived a need to develop an instrument to measure earthquakes precisely. In A.D. 132 he invented the world’s first seismograph. It was made of fine cast bronze, had a diameter of eight chi (a chi is equal to 0.237 meter), and was shaped like a wine jar. Inside the jar was a mechanism consisting of pendulums surrounded by a group of eight levers pointing in eight directions. Eight dragon figures, with a bronze ball in the mouth of each, were arranged on the outside of the seismograph. Below each dragon was a toad with mouth open upward. A strong earthquake in any direction would tilt the pendulum in that direction, triggering the lever in the dragon head. This opened the mouth of the dragon, thereby releasing its bronze ball, which fell in the mouth of the toad with a clanging sound. Thus the seismograph enabled the monitoring personnel to know both the time and direction of occurrence of the earthquake.

Galileo Galilei (1564-1642) is considered to be the founder of modern experimental science. In fact, the seventeenth century is often considered the century of genius since the foundations of modern philosophy and science were laid during that period. Galileo was inspired to study the behavior of a simple pendulum by observing the pendulum movements of a lamp in a church in Pisa. One day, while feeling bored during a sermon, Galileo was staring at the ceiling of the church. A swinging lamp caught his attention. He started measuring the period of the pendulum movements of the lamp with his pulse and found to his amazement that the time period was independent of the amplitude of swings. This led him to conduct more experiments on the simple pendulum. In Discourses Concerning Two New Sciences, published in 1638, Galileo discussed vibrating bodies. He described the dependence of the frequency of vibration on the length of a simple pendulum, along with the phenomenon of sympathetic vibrations (resonance). Galileo s writings also indicate that he had a clear understanding of the relationship between the frequency, length, tension, and density of a vibrating stretched string. However, the first correct published account of the vibration of strings was given by the French mathematician and theologian, Marin Mersenne (1588-1648) in his book Harmonicorum Liber, published in 1636. Mersenne also measured, for the first time, the frequency of vibration of a long string and from that predicted the frequency of a shorter string having the same density and tension. Mersenne is considered by many the father of acoustics. He is often credited with the discovery of the laws of vibrating strings because he published the results in 1636, two years before Galileo. However, the credit belongs to Galileo, since the laws were written many years earlier but their publication was prohibited by the orders of the Inquisitor of Rome until 1638. Inspired by the work of Galileo, the Academia del Cimento was founded in Florence in 1657; this was followed by the formations of the Royal Society of London in 1662 and the Paris Academie des Sciences in 1666. Later, Robert Hooke (1635-1703) also conducted experiments to find a relation between the pitch and frequency of vibration of a string. However, it was Joseph Sauveur (1653-1716) who investigated these experiments thoroughly and coined the word acoustics for the science of sound. Sauveur in France and John Wallis (1616-1703) in England observed, independently, the phenomenon of mode shapes, and they found that a vibrating stretched string can have no motion at certain points and violent motion at intermediate points. Sauveur called the former points nodes and the latter ones loops. It was found that such vibrations had higher frequencies than that associated with the simple vibration of the string with no nodes. In fact, the higher frequencies were found to be integral multiples of the frequency of simple vibration, and Sauveur called the higher frequencies harmonics and the frequency of simple vibration the fundamental frequency. Sauveur also found that a string can vibrate with several of its harmonics present at the same time. In addition, he observed the phenomenon of beats when two organ pipes of slightly different pitches are sounded together. In 1700 Sauveur calculated, by a somewhat dubious method, the frequency of a stretched string from the measured sag of its middle point. Sir Isaac Newton (1642-1727) published his monumental work, Philosophiae Naturalis Principia Mathematica, in 1686, describing the law of universal gravitation as well as the three laws of motion and other discoveries. Newton's second law of motion is routinely used in modern books on vibrations to derive the equations of motion of a vibrating body. The theoretical (dynamical) solution of the problem of the vibrating string was found in 1713 by the English mathematician Brook Taylor (1685-1731), who also presented the famous Taylor s theorem on infinite series. The natural frequency of vibration obtained from the equation of motion derived by Taylor agreed with the experimental values observed by Galileo and Mersenne. The procedure adopted by Taylor was perfected through the introduction of partial derivatives in the equations of motion by Daniel Bernoulli (1700-1782), Jean D Alembert (1717-1783), and Leonard Euler (1707-1783). The possibility of a string vibrating with several of its harmonics present at the same time (with displacement of any point at any instant being equal to the algebraic sum of dis- placements for each harmonic) was proved through the dynamic equations of Daniel Bernoulli in his memoir, published by the Berlin Academy in 1755. This characteristic was referred to as the principle of the coexistence of small oscillations, which, in present-day terminology, is the principle of superposition. This principle was proved to be most valuable in the development of the theory of vibrations and led to the possibility of expressing any arbitrary function (i.e., any initial shape of the string) using an infinite series of sines and cosines. Because of this implication, D'Alembert and Euler doubted the validity of this principle. However, the validity of this type of expansion was proved by J. B. J. Fourier (1768-1830) in his Analytical Theory of Heat in 1822. The analytical solution of the vibrating string was presented by Joseph Lagrange (1736-1813) in his memoir published by the Turin Academy in 1759. In his study, Lagrange assumed that the string was made up of a finite number of equally spaced identical mass particles, and he established the existence of a number of independent frequencies equal to the number of mass particles. When the number of particles was allowed to be infinite, the resulting frequencies were found to be the same as the harmonic frequencies of the stretched string. The method of setting up the differential equation of the motion of a string (called the wave equation), presented in most modern books on vibration theory, was first developed by D'Alembert in his memoir published by the Berlin Academy in 1750. The vibration of thin beams supported and clamped in different ways was first studied by Euler in 1744 and Daniel Bernoulli in 1751. Their approach has become known as the Euler-Bernoulli or thin beam theory. Charles Coulomb did both theoretical and experimental studies in 1784 on the torsional oscillations of a metal cylinder suspended by a wire. By assuming that the resisting torque of the twisted wire is proportional to the angle of twist, he derived the equation of motion for the torsional vibration of the suspended cylinder. By integrating the equation of motion, he found that the period of oscillation is independent of the angle of twist. There is an interesting story related to the development of the theory of vibration of plates. In 1802 the German scientist, E. F. F. Chladni (1756-1824) developed the method of placing sand on a vibrating plate to find its mode shapes and observed the beauty and intricacy of the modal patterns of the vibrating plates. In 1809 the French Academy invited Chladni to give a demonstration of his experiments. Napoléon Bonaparte, who attended the meeting, was very impressed and presented a sum of 3,000 francs to the academy, to be awarded to the first person to give a satisfactory mathematical theory of the vibration of plates. By the closing date of the competition in October 1811, only one candidate, Sophie Germain, had entered the contest. But Lagrange, who was one of the judges, noticed an error in the derivation of her differential equation of motion. The academy opened the competition again, with a new closing date of October 1813. Sophie Germain again entered the contest, presenting the correct form of the differential equation. However, the academy did not award the prize to her because the judges wanted physical justification of the assumptions made in her derivation. The competition was opened once more. In her third attempt, Sophie Germain was finally awarded the prize in 1815, although the judges were not completely satisfied with her theory. In fact, it was later found that her differential equation was correct but the boundary conditions were erroneous. The correct boundary conditions for the vibration of plates were given in 1850 by G. R. Kirchhoff (1824-1887). In the meantime, the problem of vibration of a rectangular flexible membrane, which is important for the understanding of the sound emitted by drums, was solved for the first time by Simeon Poisson (1781-1840). The vibration of a circular membrane was studied by R. F. A. Clebsch (1833 1872) in 1862. After this, vibration studies were done on a number of practical mechanical and structural systems. In 1877 Lord Baron Rayleigh published his book on the theory of sound; it is considered a classic on the subject of sound and vibration even today. Notable among the many contributions of Rayleigh is the method of finding the fundamental frequency of vibration of a conservative system by making use of the principle of conservation of energy now known as Rayleigh’s method. This method proved to be a helpful technique for the solution of difficult vibration problems. An extension of the method, which can be used to find multiple natural frequencies, is known as the Rayleigh-Ritz method.

In 1902 Frahm investigated the importance of torsional vibration study in the design of the propeller shafts of steamships. The dynamic vibration absorber, which involves the addition of a secondary spring-mass system to eliminate the vibrations of a main system, was also proposed by Frahm in 1909. Among the modern contributors to the theory of vibrations, the names of Stodola, De Laval, Timoshenko, and Mindlin are notable. Aurel Stodola (1859 1943) contributed to the study of vibration of beams, plates, and membranes. He developed a method for analyzing vibrating beams that is also applicable to turbine blades. Noting that every major type of prime mover gives rise to vibration problems, C. G. P. De Laval (1845-1913) presented a practical solution to the problem of vibration of an unbalanced rotating disk. After noticing failures of steel shafts in high-speed turbines, he used a bamboo fishing rod as a shaft to mount the rotor. He observed that this system not only eliminated the vibration of the unbalanced rotor but also survived up to speeds as high as 100,000 rpm. Stephen Timoshenko (1878-1972), by considering the effects of rotary inertia and shear deformation, presented an improved theory of vibration of beams, which has become known as the Timoshenko or thick beam theory. A similar theory was presented by R. D. Mindlin for the vibration analysis of thick plates by including the effects of rotary inertia and shear deformation. It has long been recognized that many basic problems of mechanics, including those of vibrations, are nonlinear. Although the linear treatments commonly adopted are quite satisfactory for most purposes, they are not adequate in all cases. In nonlinear systems, phenomena may occur that are theoretically impossible in linear systems. The mathematical theory of nonlinear vibrations began to develop in the works of Poincaré and Lyapunov at the end of the nineteenth century. Poincaré developed the perturbation method in 1892 in connection with the approximate solution of nonlinear celestial mechanics problems. In 1892, Lyapunov laid the foundations of modern stability theory, which is applicable to all types of dynamical systems. After 1920, the studies undertaken by Duffing and van der Pol brought the first definite solutions into the theory of nonlinear vibrations and drew attention to its importance in engineering. In the last 40 years, authors like Minorsky and Stoker have endeavored to collect in monographs the main results concerning nonlinear vibrations. Most practical applications of nonlinear vibration involved the use of some type of a perturbation-theory approach. The modern methods of perturbation theory were surveyed by Nayfeh. Random characteristics are present in diverse phenomena such as earthquakes, winds, transportation of goods on wheeled vehicles, and rocket and jet engine noise. It became necessary to devise concepts and methods of vibration analysis for these random effects. Although Einstein considered Brownian movement, a particular type of random vibration, as long ago as 1905, no applications were investigated until 1930. The introduction of the correlation function by Taylor in 1920 and of the spectral density by Wiener and Khinchin in the early 1930s opened new prospects for progress in the theory of random vibrations. Papers by Lin and Rice, published between 1943 and 1945, paved the way for the application of random vibrations to practical engineering problems. The monographs of Crandall and Mark and of Robson systematized the existing knowledge in the theory of random vibrations. Until about 40 years ago, vibration studies, even those dealing with complex engineering systems, were done by using gross models, with only a few degrees of freedom. However, the advent of high-speed digital computers in the 1950s made it possible to treat moderately complex systems and to generate approximate solutions in semidefinite form, relying on classical solution methods but using numerical evaluation of certain terms that cannot be expressed in closed form. The simultaneous development of the finite element method enabled engineers to use digital computers to conduct numerically detailed vibration analysis of complex mechanical, vehicular, and structural systems displaying thousands of degrees of freedom. Although the finite element method was not so named until recently, the concept was used centuries ago. For example, ancient mathematicians found the circumference of a circle by approximating it as a polygon, where each side of the polygon, in present-day notation, can be called a finite element. The finite element method as known today was presented by Turner, Clough, Martin, and Topp in connection with the analysis of aircraft structures [1].

1.2 BACKGROUND AND MOTIVATION

Some of the very prominent issues in industries such as aviation, automobiles, and defense are those concerning themselves with the issue of vibration and its effects. Vibration control has become an important aspect in improving the comfort of mechanical systems and devices which have direct human involvement. In order to achieve stability of such important systems it is necessary to understand, study and model these systems to a high degree of accurately. Several system in day to day life resemble or can be fairly approximated into several well-known vibration systems. For example, consider an artificial prosthetic leg; this is a system which closely resembles to that of a double pendulum or the suspension system of an automobile; which resembles a complex spring, mass and damper system. Thus, a deeper understanding of vibration systems cannot be overemphasized.

Early foundations of vibration systems study and modelling are laid at the undergraduate and graduate level of education. Unfortunately, the theory of vibration is perceived as one of the most abstract and difficult to understand and visualize due to the complex set of equations differential equation characterizing and governing most systems. In order to achieve greater understanding of the theory of vibration, a good balance between theory and experiments must be maintained which requires the use of different kinds of experimental set-ups. However it has been observed through extensive market research that the cost of such demonstration equipments is considerably high which limits its use especially in underdeveloped and developing countries primarily in Asia and Africa. This is the primary motivation for our investigation in this subject.

1.3 SCOPE AND OBJECTIVES

Keeping in mind the above motivation the group has developed a semi-portable and cost effective experimental set-up, keeping in mind the dearth of hardware that could exist in underdeveloped and developing countries and keeping in mind the economic limitations the implementer of such setup might face. The project was primarily divided into three major parts or cases as described below:

A. Vibration Testing of Discrete Systems

One of the most basic discrete system in physics is that of the simple pendulum. At the same time it is incompetent enough to describe all the aspects that a discrete system may exhibit and so for to be covered in the classroom. Hence, a modular pendulum set-up accommodating both the simple and compound was conceived and built. Figure 1.1 and 1.2 illustrates these systems.

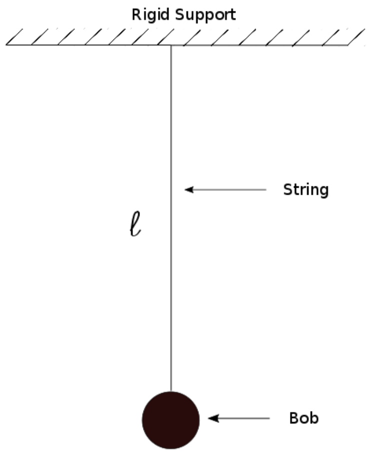


Figure 1.1: A Single Degree of Freedom System (Simple Pendulum)

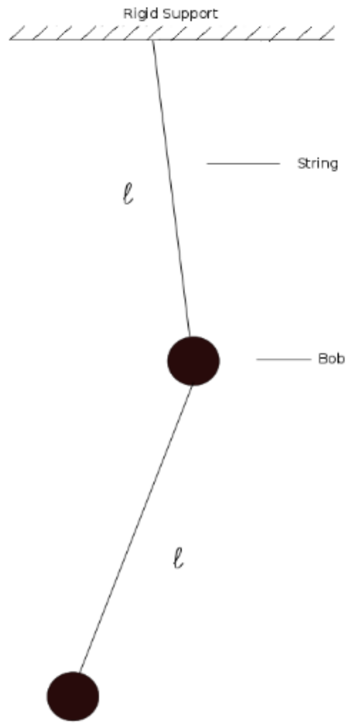


Figure 1.2: A Two Degree of Freedom System (Compound Pendulum)

B. Vibration Testing of Continuous Systems

An overhanging beam and a string under tension are the most common manifestations of the continuous degree of freedom system. However, the frequency of vibration of an overhang rod or beam may be extremely high (several hundred to thousand Hertz) due to the high modulus of elasticity and other relevant properties of this system. This could be a major source of concern especially when your aim is to design a low cost and portable set-up, as it may invite the need of some really sophisticated and expensive hardware.

Keeping in mind the above facts the team decided to go with a string under tension (fixed at both ends) as a candidate for its continuous system demonstration equipment. Figure 1.3 and 1.4 illustrates the most favorable candidates for the continuous system demonstration.

Figure 1.3: A Continuous System (Cantilever Beam)

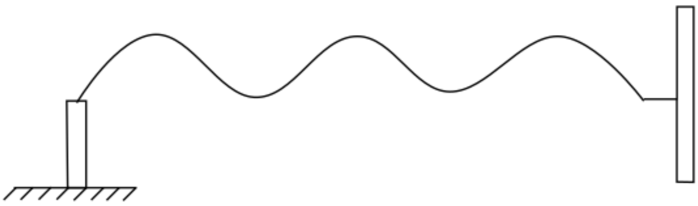


Figure 1.4: A Continuous System (String)

C. Condition Monitoring Through Vibration Signature

Building upon the experience gained in the data collection through a vision based sensor and its analysis we propose here in concept a multi-parameter condition monitoring set-up with the primary sensor being a vision based sensor.

CHAPTER 2: OVERVIEW OF VIBRATION SYSTEMS

"If you want to find the secrets of the universe, think in terms of energy, frequency and vibration" – Nikola Tesla

Part one of this chapter covers the basic concept of vibrations, different elements of a vibratory system, the concept of degree of freedom and information on discrete and continuous systems. Part two of this chapter covers the most general classification of different vibratory systems.

2.1 BASIC CONCEPT OF VIBRATIONS

2.1.1 VIBRATION:

The word Vibration comes from the Latin word *vibrationem* meaning shaking, brandishing**.** Any motion that repeats itself after an interval of time (equal or unequal) is called vibration or oscillation. The motion of a guitar string, metal scale, motion felt by passengers in an automobile traveling over a bumpy road, swaying of tall buildings due to wind or earthquake, and motion of an airplane in turbulence are typical examples of vibration. The swinging of a pendulum and the motion of a plucked string are typical examples of vibration. The theory of vibration deals with the study of oscillatory motions of bodies and the forces associated with them. The oscillations may be periodic, such as the motion of a pendulum or random, such as the movement of a tire on a gravel road.

2.1.2 PARTS OF VIBRATION SYSTEMS:

A vibratory system, in general, includes a means for storing potential energy (spring or elasticity), a means for storing kinetic energy (mass or inertia), and a means by which energy is gradually lost (damper).

The vibration of a system involves the transfer of its potential energy to kinetic energy and of kinetic energy to potential energy, alternately. If the system is damped, some energy is dissipated in each cycle of vibration and must be replaced by an external source if a state of steady vibration is to be maintained.

2.1.3 NUMBER OF DEGREES OF FREEDOM:

The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time defines the number of degrees of freedom of the system. The simple pendulum shown in Figure 1.1, as well as the torsional pendulum shown in Figure 2.1, represents a single degree of freedom system. For example, the motion of the simple pendulum can be stated either in terms of the angle or in terms of the Cartesian coordinates x and y.

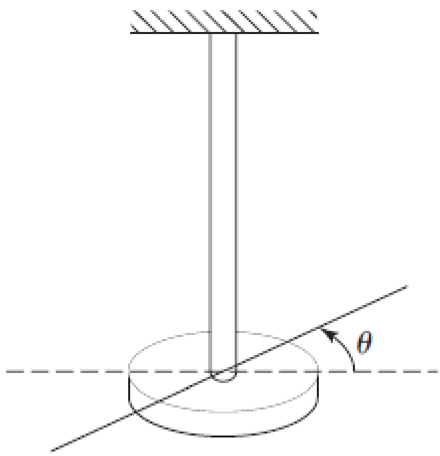


Figure 2.1: A Torsional System (Single Degree of Freedom)

If the coordinates x and y are used to describe the motion, it must be recognized that these coordinates are not independent. They are related to each other. Thus any one coordinate can describe the motion of the pendulum. In this example, we find that the choice of angle as the independent coordinate will be more convenient than the choice of x or y.

Some examples of two and three degree of freedom systems are shown from Figures 2.2 to 2.5, respectively. Figure 2.2 shows a two-mass, two-spring system that is described by the two linear coordinates $x_1$and x_2. Figure 2.3 denotes a two-rotor system whose motion can be specified in terms of θ_1 and θ_2 and hence these systems can be labelled as two degree of freedom systems.

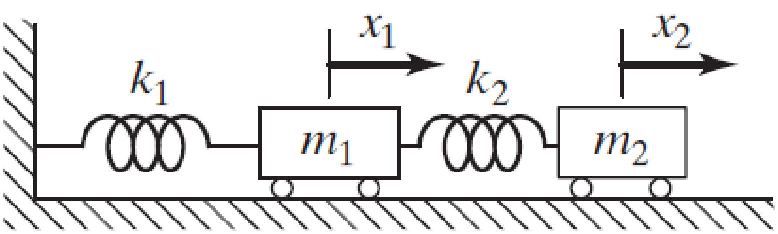


Figure 2.2: A Two Spring-Mass System (Two Degree of Freedom)

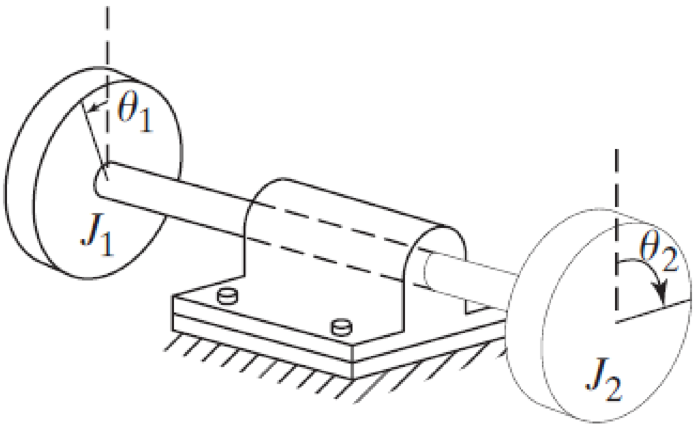


Figure 2.3: A Two-Rotor System (Two Degree of Freedom)

For the systems shown in Figures 2.4 and 2.5, the coordinatesx_i $(i = 1, 2, 3)$ and θ_i $(i = 1, 2, 3)$ can be used, respectively, to describe the motion. Hence, these systems are labelled as three degree of freedom systems.

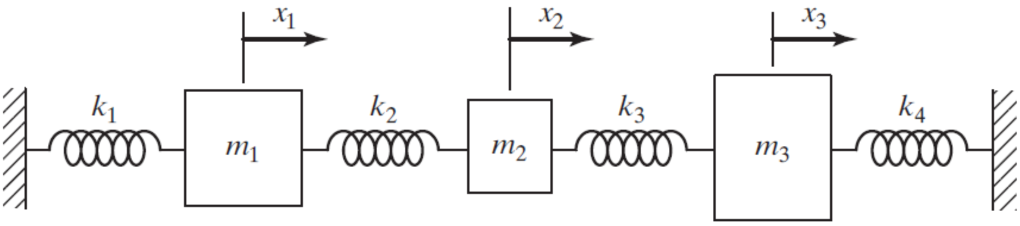


Figure 2.4: A Three Spring-Mass System (Three Degree of Freedom)

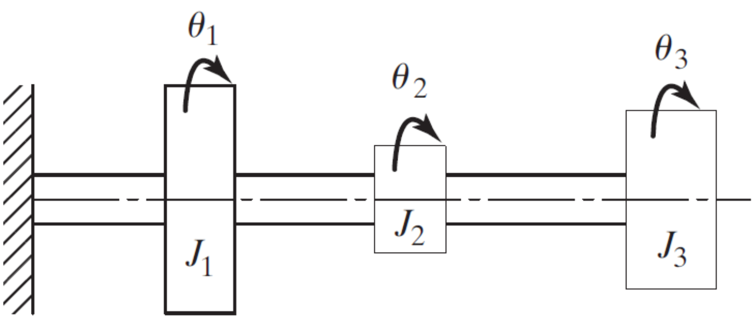


Figure 2.5: A Three-Rotor System (Three Degree of Freedom)

The coordinates necessary to describe the motion of a system constitute a set of *generalized coordinates*. These are usually denoted as $q1, q2, \ldots$ and may represent Cartesian and/or non-Cartesian coordinates.

2.1.4 DISCRETE AND CONTINUOUS SYSTEMS:

A large number of practical systems can be described using a finite number of degrees of freedom, such as the simple systems shown in Figures 2.1 to 2.5. Some systems, especially those involving continuous elastic members, have an infinite number of degrees of freedom. As a simple example, consider the cantilever beam shown in Figure 2.6. Since the beam has an infinite number of mass points, we need an infinite number of coordinates to specify its deflected configuration. The infinite number of coordinates defines its elastic deflection curve.

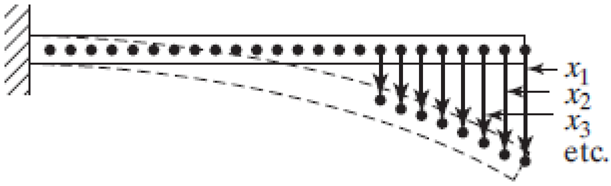


Figure 2.6: A Cantilever Beam (Infinite Degree of Freedom)

Thus the cantilever beam has an infinite number of degrees of freedom. Most structural and machine systems have deformable (elastic) members and therefore have an infinite number of degrees of freedom.

Systems with a finite number of degrees of freedom are called *discrete* or *lumped parameter* systems, and those with an infinite number of degrees of freedom are called *continuous* or *distributed* systems.

Most of the time, continuous systems are approximated as discrete systems, and solutions are obtained in a simpler manner. Although treatment of a system as continuous gives exact results, the analytical methods available for dealing with continuous systems are limited to a narrow selection of problems, such as uniform beams, slender rods, and thin plates. Hence most of the practical systems are studied by treating them as finite lumped masses, springs, and dampers. In general, more accurate results are obtained by increasing the number of masses, springs, and dampers that is, by increasing the number of degrees of freedom.

2.2 CLASSIFICATION OF VIBRATION

Vibration can be classified in several ways. Some of the important and generalized classifications are enlisted below.

2.2.1 FREE AND FORCED VIBRATION:

Free Vibration. If a system, after an initial disturbance, is left to vibrate on its own, the ensuing vibration is known as free vibration. No external force acts on the system. The oscillation of a simple pendulum is an example of free vibration.

Forced Vibration. If a system is subjected to an external force (often, a repeating type of force), the resulting vibration is known as forced vibration. The oscillation that arises in machines such as diesel engines is an example of forced vibration.

If the frequency of the external force coincides with one of the natural frequencies of the system, a condition known as *resonance* occurs, and the system undergoes dangerously large oscillations. Failures of such structures as buildings, bridges, turbines, and airplane wings have been associated with the occurrence of resonance.

2.2.2 UNDAMPED AND DAMPED VIBRATION:

If no energy is lost or dissipated in friction or other resistance during oscillation, the vibration is known as *undamped* vibration. If any energy is lost in this way, however, it is called *damped* vibration. In many physical systems, the amount of damping is so small that it can be disregarded for most engineering purposes. However, consideration of damping becomes extremely important in analyzing vibratory systems near resonance.

2.2.3 LINEAR AND NONLINEAR VIBRATION:

If all the basic components of a vibratory system the spring, the mass, and the damper behave linearly, the resulting vibration is known as *linear* vibration. If, however, any of the basic components behave nonlinearly, the vibration is called *nonlinear* vibration.

The differential equations that govern the behavior of linear and nonlinear vibratory systems are linear and nonlinear, respectively. If the vibration is linear, the principle of superposition holds, and the mathematical techniques of analysis are well developed. For nonlinear vibration, the superposition principle is not valid, and techniques of analysis are less well known. Since all vibratory systems tend to behave nonlinearly with increasing amplitude of oscillation, a knowledge of nonlinear vibration is desirable in dealing with practical vibratory systems.

2.2.4 DETERMINISTIC AND RANDOM VIBRATION:

If the value or magnitude of the excitation (force or motion) acting on a vibratory system is known at any given time, the excitation is called *deterministic*. The resulting vibration is known as *deterministic* vibration.

In some cases, the excitation is *nondeterministic* or *random*; the value of the excitation at a given time cannot be predicted. In these cases, a large collection of records of the excitation may exhibit some statistical regularity. It is possible to estimate averages such as the mean and mean square values of the excitation. Examples of random excitations are wind velocity, road roughness, and ground motion during earthquakes. If the excitation is *random*, the resulting vibration is called *random* vibration. In this case the vibratory response of the system is also random; it can be described only in terms of statistical quantities. Figure 2.7 shows examples of deterministic and random excitations.

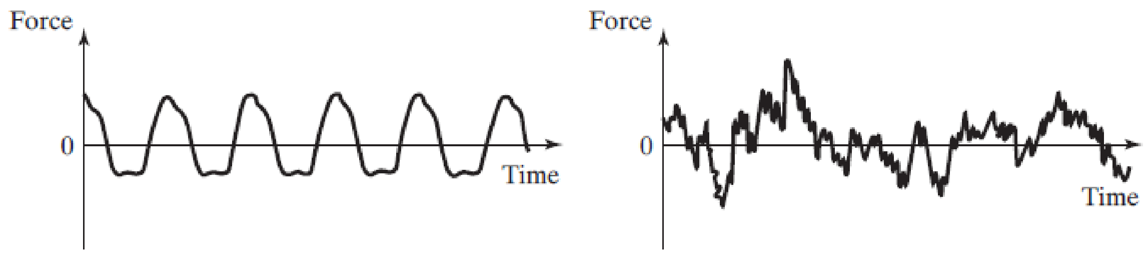


Figure 2.7: Deterministic (Left) and Non-deterministic Vibration

CHAPTER 3: LITERATURE SURVEY

This chapter enlists and describes various literatures that were referred in the due course of the project. Since our objective in undertaking this task was a much broader one that just creating a workable experimental set-up; a wide variety of literature concerning itself with the improvement of quality of demonstrations in engineering and scientific classrooms was considered. A short summary note on the literature review is presented in the last but one subdivision of this chapter which is followed by a discussion about the methods and methodologies that were finally resorted and their feasibility in the eyes of the authors.

3.1 LITERATURE REFERRED FOR SIMPLE PENDULUM

A Belendez el at in their paper have discussed the nonlinear oscillation of a simple pendulum and presents not only the exact formula for the period but also the exact expression of the angular displacement as a function of the time, the amplitude of oscillations and the angular frequency for small oscillations. The angular displacement is written in terms of the Jacobi elliptic function $sn(u;m)$ using the following initial conditions: the initial angular displacement is different from zero while the initial angular velocity is zero. The angular displacements were plotted using Mathematica, an available symbolic computer [2]. Other works on analytical/theoretical solutions can be found in [3] [4] [5]

Robert Nelson et al have discussed comprehensively the use of a plane pendulum to measure acceleration due to gravity to four significant figures in a simple laboratory experiment. In their study the domain over which the time period for oscillations is measured is increased to 100 oscillations and several corrections such as the finite amplitude correction, mass distribution correction , wire corrections et cetera were employed to better understand the pendulum system [6].

E K Dunn has studied the effect of string drag on the pendulum motion. He in his study using a HeNe laser and a photodiode to measure the time period has reported a string drag of 5%, which is small but considerable [7].

Akhil Arora et al have used the oscillatory motion of a suspended bar magnet to throws light on the damping effects acting on the pendulum. The viscous drag offered by air was found to be the main contributor for slowing the pendulum down. The nature and magnitude of the damping effects was shown by them to be strongly dependent on the amplitude [8].

Mostafa Abdelkader in his article has considered a simple pendulum acted on by gravity and subjected to a resistance proportional to the velocity of the bob. He states further in his article that if the length of the string and the mass of the bob are held constant, the amplitude of the bob decreases gradually because of the damping. But if we want to keep the maximum swing of the bob constant for all time; we achieve this by varying the length of the string, the mass of the bob or both [9].

Cesar Medina et al in their article on experimental control of simple pendulum have considered the quantitative analysis of systematic error that ‘model assumptions’ pose in the analysis of simple pendulum in practice. Their quantifying parameters such as ‘small initial angle’, ‘point mass’ et cetera have led to a deeper comprehension of the simple pendulum system [10].

Michael Fowler in an effort to understand the non-linear differential equation of the simple pendulum a little better has used Excel® spreadsheets to solve the differential equation by employing numerical differentiation procedures. This work was aimed at high school students though in an effort to make them understand the physics of the pendulum a little better [11].

Vadas Gintautas et al in their paper have used a ball-type computer mouse to demonstrate the measurement of both the frequency and coefficient of damping of a simple physical pendulum. This easily constructed laboratory equipment makes it possible for all students to have hands-on experience with one of the most important simple physical systems [12].

David Auslander in his article on animations in physics discusses its importance in pedagogical work. He has used fundamental equations of motions of the simple pendulum with angle approximations to create simple animations in MATLAB® [13]

Gulf Coast Data Concepts in its article on 'Calculating the Acceleration of Gravity Using an Accelerometer Data Logger' has used a USB Accelerometer to obtain acceleration versus time graph of the pendulum bob (The USB data logger constitutes a substantial part of the pendulum bob). Displacement and velocity versus time graphs can also be found using numerical integration techniques. The readings from the accelerometer were analyzed using a range of different numerical techniques and several interesting plots were obtained [14].

R Kavithaa et al have presented a vision based approach on the analysis of the simple pendulum which is by far the most inspiring article for the work on this project. An oscillating simple pendulum with mass attached to a thread and rigidly supported from other end was captured in a video file using a camera, which was further processed in OpenCV to obtain the time period of oscillations/vibrations [15].

Bilal Usman et al have done a similar study to the above reference, using a sophisticated camera and employing MATLAB® for analysis. The collected data was also verified using a phone accelerometer data [16].

W Kwong Wong et al in their article on pendulum experiments with three modern electronic devices have made use of an Arduino based accelerometer, a smartphone and a LEGO® Mindstorm NXT to study the dynamics of simple pendulum [17].

Another method to find the characteristics of the pendulum was done using a potentiometer mounted as a pivot about which the pendulum oscillates [18]. This system is elaborated further in section 2.3.

In place of a potentiometer an encoder can be used to stud the angular displacement of the pendulum shaft as it has a lesser damping effect on the pendulum. High accuracy encoders give minute to negligible errors and have a better life as compared to that of potentiometers [19].

Another form of non-intrusive technique is the use of a photogate. A photogate is a sensor consisting of an infrared LED and a photodiode. Whenever the contact between the LED and the photodiode is broken by any object (in this case the pendulum bob) the system is able to detect it. This principle is used to study the pendulum motion in. However this system requires a large number of photogates in order to accurately represent the system [20] the application of photoelectric timing circuits using a similar methodology to that of the above reference can be obtained in [21] [22].

Some interesting simulations on the simple pendulum can also be found in the following articles [23] [24].

3.2 LITERATURE REFERRED FOR DOUBLE PENDULUM

M Z Rafat et al have investigate a variation of the simple double pendulum in which the two point masses are replaced by square plates. The double square pendulum exhibits richer behavior than the simple double pendulum and provides a convenient demonstration of nonlinear dynamics and chaos. It is also an example of an asymmetric compound double pendulum, which has not been studied in detail. They have obtained the equilibrium configurations and normal modes of oscillation and derived the equations of motion, which are solved numerically to produce Poincare sections. They have also shown how the behavior varies from regular motion at low energies, to chaos at intermediate energies, and back to regular motion at high energies. Also the onset of chaos occurs at a significantly lower energy than for the simple double pendulum [25] a similar work with some additional simulations can be found in [26].

Roja Nunna in her article on the numerical analysis of the planar double pendulum has numerically analyze the dynamics of the double pendulum system. First, the physical system is introduced and a system of coordinates is fixed, and then the Lagrangian and the Hamiltonian equations of motions are derived. She has further indicated that the system is governed by a set of coupled non-linear ordinary differential equations and using these, the system can be simulated. Finally this article analyzes Poincaré sections, the largest Lyapunov exponent, progression of trajectories, and change of angular velocities with time for certain system parameters at varying initial conditions [27].

Wenhua Hai et al in their article on pendulum chaos have studied a compound pendulum with deterministically periodic perturbation. In their analytical approximation, chaotic solution initially near the homoclinic one is constructed and its boundedness conditions are established. It is shown that the chaotic solution is analytically bounded and numerically unbounded, which describes a non-periodical vibration around unstable equilibrium of the corresponding unperturbed system [28] Study of chaos of the double pendulum with simulations can also be found in [29].

Danny Hoskin in his mathematics project on pendulums has begun by the study of a simple pendulum and discussion about the motion. The project continues by deriving the equations of motion for the double pendulum, and a study of the motion governed by these equations here he has introduced the idea of quasiperiodic motion. The project then tackles the system when the pivot vibrates, and is shown that the pendulum can be maintained in an 'upside-down' position with this vibration. The mathematical analysis of the unusual stability of inverted/ upside-down pendulum is also presented [30].

Arnab Dhabal et al in their project thesis have built a double pendulum and demonstrated chaotic and non-chaotic evolution of the system based on different initial conditions. A numerical treatment of an ideal undamped double pendulum is also done to verify experimental results [31].

Daniel Kelly in his article describes the design, fabrication, and testing of an apparatus that accommodates several different pendulum designs. The experiment uses a 1hp motor to drive a 0.9761kg single pendulum and 1.573kg double pendulum in a sinusoidal manner. A high-speed camera records the motion and a digital image analysis program extracts data. Initial tests show that a single pendulum driven at 1.79Hz at an amplitude of 5.40cm displays period doubling behavior. Also, repeated experiments on the decay of an un-driven double pendulum demonstrate sensitivity to initial conditions: the trajectories quickly diverge from one another even though the initial conditions are nearly the same [32].

O K Ukoba et al in their paper have classified state of the art tracking methods and used vision based method to do motion analysis of a double pendulum using MATLAB®. They further conclude in their study that for large motions the double pendulum is a chaotic system, but for small motions it is a simple linear system. For small angles, a pendulum behaves like a linear system. When the angles are small in the Double Pendulum, the system behaves like the linear Double Spring. Also, for large angles, the pendulum is non-linear and the phase graph becomes much more complex [33].

R Nielsen et al in their thesis on the double pendulum have established the regimes of chaos in the double pendulum. This was done using experimental as well as computer simulation. Experimental study was done using a high fps camera (400 fps) and was then processed with MATLAB®. Classical Lagrangian and Hamiltonian equations were used for computer simulations. The computer simulations were successfully compared to the experimental data [34].

3.3 LITERATURE REFERRED FOR CONTINUOUS DEGREE OF FREEDOM SYSTEMS

A Systems which have a finite number of degrees of freedom are known as discrete or lumped parameter systems, and those systems with an infinite number of degrees of freedom are called continuous or distributed systems. Analysis of structures with discrete coordinates is easy and a practical approach for analysis of structures with dynamic loads, however the results obtained are approximate values and not accurate ones. However analyses of a continuous system is more reliable and gives more accurate solution. There may be complexities but analysis of a continuous system gives solutions showing actual behavior of dynamic structures.

.

This study illustrates various methods used to measure the displacement at different nodes in a structure undergoing forced vibration. The structure used is a cantilever beam .A beam with fixed support at one end with no support at the other end is called a cantilever beam. The beam is excited using shaker or by giving small initial displacement at the tip. The material selected for the experiment is Aluminum.

Many methods are available for the measurement of displacement, some of them include use of accelerometer, strain gauge, Laser Doppler Vibrometer (LDV), High Speed Electronic Speckle Pattern Interferometry (ESPI), ZnO piezoelectric sensor, Linear Variable Differential Transformer (LVDT], etc. However the most common method of vibration analysis is the use of accelerometer and data acquisition system.

Nikos Kiritsis et al in their paper on multi-purpose vibration experiment using LabVIEW® have studied the response of an aluminum cantilever beam under harmonic excitation. The measurements are simultaneously measured using a strain gage, a linear variable differential transformer and an accelerometer, and compared with the real time theoretical response. All data acquisition and analysis is done using a custom built LabVIEW virtual instrument. This fundamental experiment from the vibration area was used at McNeese State University in many different ways throughout the mechanical engineering curriculum. Vibration measurement using Accelerometer involves the Accelerometer probe, connecting cables, data acquisition system, Oscilloscope. Accelerometer produces voltage signals related to the displacement caused by the vibrations. These electric signals are then transferred to the data acquisition system where they are converted into the functional data and output is in the form of FFT graph. Cantilever beam is mounted on a table with an exciter are also studied using two linear variable differential transformers (LVDTs) [35].

Strain Gauges are also used in vibration measurement filed. Strain gauges give response to vibrations in terms of electrical signal. Due stresses developed in the specimen because of vibrations the resistance of the strain gauge is changed and can be measured with the help of Whetstone Network. Using gauge factor resistance recorded is converted into strain data. With the help of various computer aided software this strain data is converted into relative displacement and then results are generated [36].

A B Stanbridge et al in their paper on modal testing using laser doppler vibrometers have studied a number of vibration mode shape of a sinusoidally excited structure. The LDV vibration output is an amplitude-modulated sine wave and mode shapes, defined along the scan line, may therefore be established by demodulation. Alternatively, in the frequency domain, the LDV output is a line spectrum, with sidebands centered on the excitation frequency and spaced at the scan frequency. Laser Doppler Vibrometer are also used for the vibration measurement of continuous systems which is having advantage of contactless measurement. LDV apparatus involves Laser Transmitter, beam splitter, oscillating mirror and photodiode. Laser beam from transmitter is passed through the beam splitter and divided into Testing Beam and Reference beam. Testing beam is then focused on the vibrating specimen and the response beam is captured. The Response beam and reference beam are then tested for the Doppler shift. Magnitude of Doppler shift gives the relative displacement at that particular point [37].

Mateusz Romaszko et al discuss that, mounting an accelerometer with all it signal cables may cause considerable system modifications and introduce a high degree of damping. A better and non-intrusive technique was explored by using high speed video analysis. In this method a video footage of the oscillating bob was captured at a 1000 frames per second. This was then imported into a suitable image processing software like MATLAB® and analyzed to extract the motion dynamics of a oscillating cantilever beam [38] another interesting study employing CCD cameras can be found in [39].

B R Jooste et al in their article on the theoretical study of vibrations of a cantilever beam using ZnO piezoelectric sensor present that piezoelectric sensors can measure vibrations of solid structures very accurately. A model of a cantilevered beam, with a ZnO film on one side was presented by them in their study. Both viscous and internal damping are considered. The output of the sensor was modeled and matched with experimental results by adjusting the damping parameters. A theoretical formulation for damage was introduced. Experimental results for a damaged beam confirm the shift in frequencies to lower values and thus the vibrations for the beam were studied [40].

Note:

Piezoelectric materials are materials that produce an electric current when they are placed under mechanical stress or vice a versa. Piezoelectric sensors in the form of a thin film could potentially be used as a sensing device by directly measuring the voltage output generated by the strained film. The piezoelectric properties of ZnO film enable it to act as a sensor when the film experiences strain, a voltage differences is created across the film. This film is developed by magnetron sputtering method. A ZnO thin film was developed on the cantilever beam to study its vibration properties.

Jacek Snamina et al have studied the vibrations of a cantilever beam with magnetorheological fluid by using the acoustic signal. The study was to determine the changes in the acoustic field around the vibrating beam caused by modifications of the MR fluid properties resulting from the changes of external magnetic field strength [41].

H Van der Auweraer et al have discussed the various critical elements of a modal testing system based on pulsed-laser holographic ESPI measurements. Such system allows making very high spatial resolution measurements on panel-like structures at frequencies that are of relevance for the vibroacoustic behavior [42] a similar study can be found in [43].

3.4 <u>LITERATURE REFERRED FOR VIBRATION SIGNATURE CONDITION MONITORING</u>

A preliminary literature review exposed us to the various available developments in the field of Machine Fault Signature Analysis. There is a huge scope and critical importance of this particular application of Vibration sensing in our modern world of automated machinery and optimum designs. Various types of faults like bearing faults, coupling faults, electrical machinery faults, transmission system faults, etc. have been studied by studying their vibration signatures. The basic underlying concept here follows that a normally operating machine shows a constant vibration, however, any anomaly observed in the vibration sensing indicates a fault in the machine.

Moheadin Yousefi has used approaches from numerical, analytical, FEM and experimental approaches exist, involving modelling a cracked beam on ANSYS, finding its fundamental frequency and verifying the results mathematically [44], as well as structural health monitoring using accelerometers and force gauge with heavy data processing and mathematical modelling[45].

A few other approaches were referred to such as using Fast Fourier Transforms on the time domain vibration signal of a faulty bearing obtained by using sophisticated equipment, to be used as a knowledge base for future analysis and fault prediction. This method, however, gives incoherent results and the need for further development using other methods was suggested by the author [46].

Diagnostic Testing by Vibration Analysis has been successfully pursued since as early as 1996, where 93 identical high-voltage circuit-breakers were tested for faults using accelerometers and data acquisition systems during field testing. Diagnosis of deviations in these vibrations accurately recommended requirement of re-lubrication, or inspection of a few circuit-breakers. Upon inspection, certain deviations in positioning of internal components was observed, indicating the effectiveness of vibration analysis methods for fault finding [47].

Yongzhi Qu et al adopted a novel approach to identify the faults in aluminum plate and blocks by generating an impulse excitation signal by an impact hammer and measuring the vibration response of the structures due to the impulse by employing Acoustic Emission (AE) Sensors. An appropriate signal processing method is used to identify the damage by comparing the detectable changes Frequency Response Functions (FRFs) of healthy and damaged structures.

This study indicated that changes in modal parameters due to damage may be used for fault detection [48].

Dr. Marko Morbidini at the Imperial College London extensively demonstrates an experimental study of Thermosonics and vibroacoustic modulation (VAM) NDT techniques to detect fatigue cracks in metals and other types of interface defects. These two techniques are capable of detecting very small fatigue cracks and impact damage in structures, which find applications in the aerospace and nuclear power industries. The study also suggested that Thermosonics has a higher sensitivity to detect cracks as compared to VAM. Such reliable damage detection contribute to cost savings in the maintenance sector of various modern-day industries. Ultrasonic and thermal NDT Techniques are a few other approaches which have been studied [49].

3.5 SUMMARY OF LITERATURE SURVEY

It was found that many of the above systems used methods which were cost intensive and required a high degree of skill to operate. Also, they had a considerable maintenance requirement. Vision based techniques were being increasingly implemented to study various physical/mechanical systems.

3.6 DISCUSSION

As the emerging trend of the vision based method in studying various physical and mechanical systems is apparent. The method is highlighted in the below discussion.

A. Vision Based Method

Contactless methods are currently dominant in the measurement of vibrations of continuous systems. They perform particularly well in the analysis of vibrations of so-called lightweight constructions, in which the application of conventional measuring systems utilizing e.g. accelerometers is not viable. As discussed earlier Accelerometers introduce additional mass, which causes changes in both damping and rigidity of the studied system.

Measurements using the vision method is done in three stages:

- Selection of hardware
- Recording of the sequence of images by a camera
- Development of an algorithm using a suitable software

The first stage encompasses analysis of the measuring task and selection of hardware configuration of the vision system. The optical system is selected, taking into account the dimension of object to be studied. The image is set up so that the entire object and other moving element are visible.

The second stage of measurements using the vision method involves the recording of a sequence of object vibration images at the experimental set-up.

The third stage of measurements of the object vibrations using the vision method involves the development of an algorithm for analysis of the recorded sequence of images. This algorithm can be developed in software like MATLAB/SCILAB/OpenCV. The purpose of the analysis is to determine the displacements of each of the points visible on the image defining the amplitude of vibrations observed for each of the three frequencies. The recorded images are subjected to processing. Elimination of noise and accidental flaws in form of reflexes observed on the image is done during processing through .The final step of the third stage of measurements using the vision method is the processing and saving of the results of measurements.

Myriad of researches are being done in the field of vibration measurement with the latest one being the vision based method. However all of the above methods are expensive and the cost of their apparatus is beyond the reach of some of the universities.

B. Reasons for choosing Vision based method:

- It is a contactless method, hence there is no problem of damping arising as well as there is no addition of self-weight of the instrument to the apparatus.
- Other methods are complex and include a lot of calculations which are time consuming as well as error prone
- Maintenance cost is low
- Higher rate of accuracy
- Less time consuming once the technique is well established for a given set of conditions
- It does not require skilled manpower for use after development

- Can be used to analyze vibrations of very high frequency with the help of a sophisticated sensor

CHAPTER 4: THE OSCILLATING PENDULUM

…. I had along with me….the Descriptions, with some Drawings of the principal Parts of the Pendulum-Clock which I had made, and as also of them of my then intended Timekeeper for the Longitude at Sea.

John Harrison

4.1 INTRODUCTION

To this day 400 years after Galileo the pendulum is as surprising as ever. Simple pendulums have been used in time keeping since olden days and were also used to measure acceleration due to gravity ‘g’ at various places on and above the earth. Galileo and Aristotle both studied the simple pendulum extensively to investigate its nature further. Galileo Galilei (1564-1642) first introduced the concept of a simple pendulum when he was watching a lamp in a church oscillate (it was his responsibility to see that it kept on burning). He was a medical student at that time, so he used his pulse to record the time taken for the oscillations. He found that, contrary to common belief, the time taken by the pendulum for each oscillation is the same independent of the amplitude [50]. Early sailor used the pendulum as a primary mean of timekeeping.

The pendulum can be and was regarded as the standard of time, down to the first part of this century. Indeed, in improved and refined forms, the pendulum has been used not only in terrestrial but also in astronomical measurements such as determining whether in fact the rotation of the earth about its own axis is uniform. This particular use of the pendulum is sufficient in itself to show the importance of the pendulum in science [52].

While the treatments of pendulum motion in the university textbooks differ from each other in some details the basic sequence of the argument is the same in each. For most the mechanics of pendulum motion was related closely to an understanding of motion down an inclined plane and treatment of the mechanics of motion on inclined planes precedes the analysis of pendulum motion [53]. Pendulums come in all shapes, sizes, forms and structures. A brief account of different pendulums can be obtained in book ‘Understanding Pendulums’ [54].

Michael Matthews in his book ‘Time for Science Education’ has written beautifully on how the teaching the history and philosophy of pendulum motion can contribute to science literacy [51] Yunjie Miao et al in their paper on resonant frequency analysis of artificial lower limbs have applied the compound pendulum model to analyze the swing motion during walking for the wearer and the exoskeleton [55].

4.2 THE SIMPLE PENDULUM

The following section gives a detailed account of the simple pendulum experiment and its comparison with theoretical response.

4.2.1 EQUATIONS OF MOTION:

As we can point out previously, one of the simplest nonlinear oscillating systems is the simple pendulum. This system consists of a particle of mass m attached to the end of a light inextensible rod, with the motion taking place in a vertical plane. The differential equation modelling the free undamped simple pendulum is

$$\frac{d^2\theta}{dt^2} + {\omega_0}^2 \sin\theta = 0 \tag{1}$$

ω_0 is defined as

$$\omega_0 = \sqrt{\frac{g}{l}} \tag{2}$$

We consider that the oscillations of the pendulum are subjected to the initial conditions

$$\theta(0) = \theta_0 \qquad \left(\frac{d\theta}{dt}\right) = 0 \qquad \text{at } t = 0 \tag{3}$$

Equation (1), although straightforward in appearance, is in fact rather difficult to solve because of the nonlinearity of the term $sin\theta$. In order to obtain the exact solution of Equation (1), this equation is multiplied by $d\theta/dt$, so that it becomes

$$\frac{d\theta}{dt}\frac{d^2\theta}{dt^2} + {\omega_0}^2 \sin\theta \frac{d\theta}{dt} = 0 \tag{4}$$

$$\frac{d}{dt}\left[\frac{1}{2}\left(\frac{d\theta}{dt}\right)^2 - {\omega_0}^2 \cos\theta\right] = 0 \tag{5}$$

Equation (5), which corresponds to the conservation of the mechanical energy, is immediately integrable, taking into account initial conditions in Equation (3). From Equation (5) we can obtain

$$\left(\frac{d\theta}{dt}\right)^2 = 4{\omega_0}^2\left[\sin^2\left(\frac{\theta_0}{2}\right) - \sin^2\left(\frac{\theta}{2}\right)\right] \tag{6}$$

$$\cos\theta = 1 - 2\sin^2\left(\frac{\theta}{2}\right) \tag{7}$$

Now let

$$y = \sin(\theta/2) \tag{8}$$

and

$$k = \sin^2\left(\frac{\theta_0}{2}\right) \tag{9}$$

Therefore we have

$$y(0) = \sqrt{k} \tag{10}$$

It is easy to obtain the value of $d\theta/dt$ as a function of dy/dt as follows. Firstly,

$$\frac{dy}{dt} = \frac{dy}{d\theta}\frac{d\theta}{dt} = \frac{1}{2}\frac{d\theta}{dt}\cos\left(\frac{\theta}{2}\right) \tag{11}$$

And secondly

$$\left(\frac{dy}{dt}\right)^2 = \frac{1}{4}\cos^2\left(\frac{\theta}{2}\right)\left(\frac{d\theta}{dt}\right)^2 = \frac{1}{4}[1-\sin^2(\theta/2)]\left(\frac{d\theta}{dt}\right)^2 = \frac{1}{4}(1-y^2)\left(\frac{d\theta}{dt}\right)^2 \tag{12}$$

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{4}{1-y^2}\left(\frac{dy}{dt}\right)^2 \tag{13}$$

Solving further and substituting

$$\frac{4}{1-y^2}\left(\frac{dy}{dt}\right)^2 = 4{\omega_0}^2(k-y^2) \tag{14}$$

$$\left(\frac{dy}{dt}\right)^2 = {\omega_0}^2 k(1-y^2)\left(1-\frac{y^2}{k}\right) \tag{15}$$

We do define new variables τ and z as

$$\tau = \omega_0 \mathrm{t} \quad \text{and} \quad z = \frac{y}{\sqrt{k}} \tag{16}$$

$$\left(\frac{dz}{d\tau}\right)^2 = (1-z^2)(1-kz^2) \tag{17}$$

$$z(0) = 1 \qquad \left(\frac{dz}{d\tau}\right) = 0 \qquad at\ \ \tau = 0 \tag{18}$$

Therefore

$$d\tau = \pm\frac{dz}{\sqrt{(1-z^2)(1-kz^2)}} \tag{19}$$

Integrating

$$\tau = -\int_1^z \frac{d\zeta}{\sqrt{(1-\zeta^2)(1-k\zeta^2)}} \tag{20}$$

$$\tau = \int_0^1 \frac{d\zeta}{\sqrt{(1-\zeta^2)(1-k\zeta^2)}} = \int_0^z \frac{d\zeta}{\sqrt{(1-\zeta^2)(1-k\zeta^2)}} \tag{21}$$

which allows us to obtain τ as a function of z and k as

$$\tau(z) = K(k) - F(\arcsin z; k) \tag{22}$$

where K(m) and F(ϕ;m) are the complete and the in- complete elliptical integral of the first kind, defined as follows

$$K(m) = \int_0^1 \frac{dz}{\sqrt{(1-z^2)(1-mz^2)}} \tag{23}$$

$$F(\varphi; m) = \int_0^\varphi \frac{dz}{\sqrt{(1-z^2)(1-mz^2)}} \tag{24}$$

The period of oscillation T is four times the time taken by the pendulum to swing from θ = 0 (z = 0) to $\theta \ = \ \theta_0$ (z = 1). Therefore

$$T = 4t(0) = \frac{4\tau(0)}{\omega_0} = \frac{4}{\omega_0}K(k) = \frac{2}{\pi}T_0 \mathrm{K(k)} \tag{25}$$

$$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{l}{g}} \tag{26}$$

$$F(\arcsin z; k) = K(k) - \tau \tag{27}$$

which can be written in terms of the Jacobi elliptic function $sn(u; m)$

$$z = sn\ (K(k) - \tau; k) \tag{28}$$

$$\sin\left(\frac{\theta}{2}\right) = \sin\left(\frac{\theta_0}{2}\right) sn\ \left[K\left(\sin^2\frac{\theta_0}{2}\right) - \omega_0 \mathrm{t}\,;\ \sin^2\frac{\theta_0}{2}\right] \tag{29}$$

$$\theta(t) = 2\arcsin\left\{\sin\frac{\theta_0}{2} sn\ \left[K\left(\sin^2\frac{\theta_0}{2}\right) - \omega_0 \mathrm{t}\,;\ \sin^2\frac{\theta_0}{2}\right]\right\} \tag{31}$$

$$\omega(\theta_0) = \frac{\pi\omega_0}{2\mathrm{K}(\sin^2[0.5*\theta_0])} \tag{32}$$

Equation (31) corresponds to a simple harmonic oscillator, but with the following exact expression for the angular frequency

$$\omega(\theta_0) = \frac{\pi\omega_0}{2\mathrm{K}(\sin^2[\theta_0/2])} \tag{33}$$

4.2.2 THE EXPERIMENT:

Image processing technique was used to gather data about motion of simple pendulum against a monochrome background under well illuminated laboratory conditions and the result were compared with theoretical response.

A. Experimental Setup

The apparatus consisted of a bob having a smooth surface (mass 46 g, diameter 43 mm), string (nylon, length 2.05 m), Vernier caliper, web camera, computer and a stopwatch (Figure 2). The bob was colored black in order to facilitate easier and accurate detection. The radius of the bob is determined using the Vernier caliper so that the length to the center of mass of the bob (ℓ) can be accurately determined (ℓ = length of string + radius of bob). The setup was illuminated by fluorescent light. It was isolated from external disturbances such as wind and sudden changes in illumination conditions. A stopwatch was used to validate the time period obtained using image processing technique.

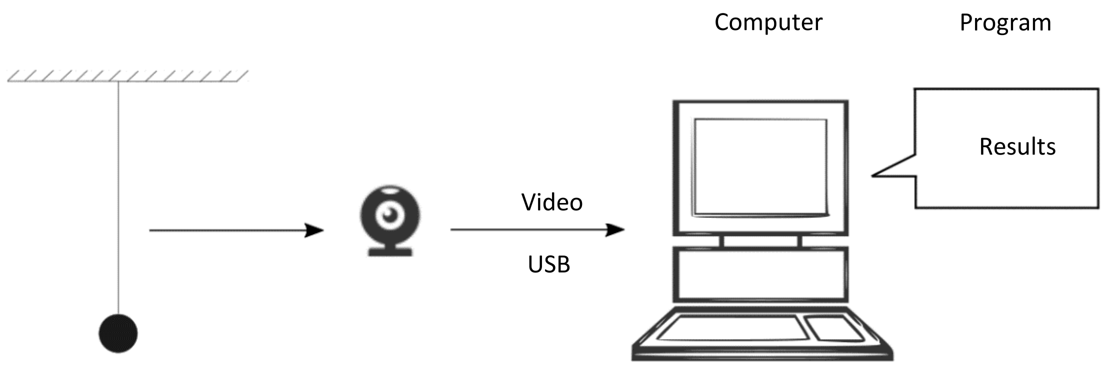


Figure 4.1: Schematic Representation of Experimental Setup

B. Details of Image Processing Sensor

An inexpensive USB web camera (iBall® ROBO K20) was used to capture a video recording of an oscillating bob at 18 frames per second (fps) with a resolution of 160×140 pixels. The camera could focus on objects at a distance of 5 cm to infinity. The camera was placed at a distance of 150 cm from plane of oscillation of the pendulum. The vertical position of camera was kept approximately equal to that of the bob.

C. Procedure

The video clip of the oscillating pendulum was imported in MATLAB®. The video stream was processed by a program developed to obtain the coordinates of centroid of the bob image using an algorithm adapted from [18]. These coordinates were then converted into global metric coordinates by pixel calibration using Eq. (1).

$$D^g = \kappa \times D^p \tag{34}$$

The time taken for one oscillation of the bob was verified using a stopwatch. A good approximation of the time period for an undamped pendulum can also be found out using Eq. (2) which is applicable for small amplitudes[1].

[1] When the approximation $\sin\theta \approx \theta$ can be made with a satisfactory degree of accuracy

$$Time\ period(T) = 2\pi\sqrt{\frac{\ell}{g}} \tag{35}$$

Objectives:

- Excite the system to get the free vibrations.
- Capture the displacement data with respect to time using motion camera,
- Generate a MATLAB code to get the Displacement vs Time graph using pictorial data captured by the camera.
- Match the experimental output with the theoretical calculations.

Experimental Set-up:

- Four rods are fixed at the definite positions to support the system.
- Elastic string is fixed at the one end and it's another end is supported over the pulley with the help of dead weights (Masses of 20g,40g or 60g).
- Tension in the string can be varied by using different masses.
- Effective length of the string is measured (The length between the fixed point and the point of contact between string and the pulley).
- Mass per unit length, ρ and the tension inside the string, P are measured.
- The points are marked on the string where we have to find the displacement with respect to time. Points are marked red so that they can be easily captured against the white background.
- Distance of the marked points from the x=0 position, is measured.
- The motion camera and focusing LEDs are placed at the definite position to capture the vibrations.

Specifications of camera:

- Logitech webcam c920 HD PRO camera
- Full HD glass lens
- Full HD video recording (up to 1920 x 1080 pixels)
- Automatic low-light correction
- Built-in dual stereo mics with automatic noise reduction

Procedure:

- String is excited through the particular force and set to vibrate freely. Limited forced is applied so that string can initially vibrate at fundamental mode i.e. 1^{st} mode of vibration.
- Camera is turned on to capture the vibrations of the string. Camera focuses on the points marked on the string at the definite distance. Camera used for the experiment has 30 fps capacity i.e. it generates 30 frames per seconds. Each frame records the vibrations of different points on the string at different time.
- This data is then transferred to the computer attached to the camera. Then this data is processed using MATLAB software to generate the graphical output.
- Displacement vs Time Graph is plotted for the complete length of the string. Graph shows the displacement of various points on the string with respect to time.
- Graphical data is then cross-checked with the theoretical calculations for the conformance to the output.
- Procedure can be repeated for different lengths of the string to get the variations in the output. Also procedure can be repeated for different tensions in the string. The tension can be varied using different mass of dead weight.

4.2.3 EXPERIMENTAL RESULTS:

The MATLAB® program implemented was used to extract global position co-ordinates of center of bob against time. From this data, displacement versus time curve was obtained. The velocity Equation 3 versus time and acceleration versus time curves (Figure 3) were obtained by numerically differentiating displacement and velocity curve by using central difference method respectively.

$$v = \frac{dx}{dt} = \frac{x_{n+1} - x_{n-1}}{2\,\Delta t} \tag{16}$$

In the present study Δt = 0.25 seconds.

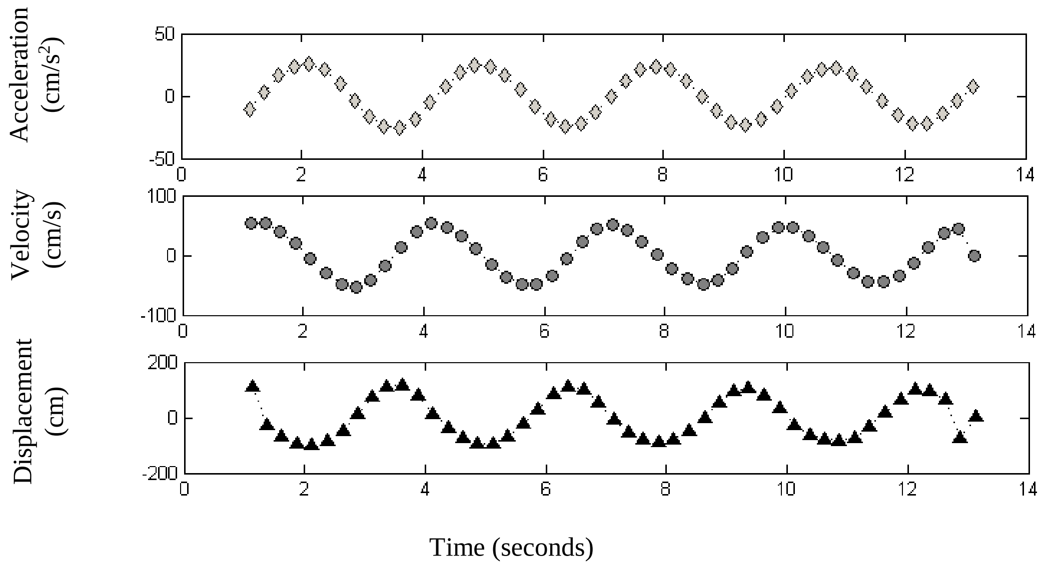


Figure 4.2: Comparison of Experimental Results (Simple Pendulum)

The maximum distance moved by bob from its mean position to extreme position is called its Amplitude (X_0) or peak amplitude of oscillation. This was calculated using Eq. (4).

$$Amplitude(X_0) = \max_{i=1}^{i=n}\left(D_i^g - \frac{D_1^g + D_2^g + \cdots + D_n^g}{n}\right) \tag{37}$$

where n equals number of frames captured and D^g is the distance in pixels in global metric coordinates as described previously in Eq. (1).

The theoretical response of a damped single degree of freedom system is given by Eq. (5). The damping coefficient (ξ) of the system was calculated using Eq. (5) and (7). A plot of undamped theoretical displacement $x(t)$ Eq. (6), damped theoretical displacement $x_d(t)$ and experimental results is shown in Figure 4.

$$x_d(t) = X_0\, e^{-\xi t} \sin(\sqrt{1-\xi^2}\, \omega_n\, t) \tag{38}$$

$$x(t) = X_0 \sin(\omega_n\, t) \tag{39}$$

$$\frac{1}{n}\ln\left(\frac{x_1}{x_{n+1}}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \tag{40}$$

where x_1 is the amplitude at starting position and x_{n+1} is the amplitude after n cycles.

The deviation of experimentally obtained displacement data and theoretical solution to damped system was found to lie between ±7 percent which is acceptable for the present work.

The setup used in this method costed less than ₹1000 which is significantly low as compared to other commercial setups available in the market. The major cost incurred was that of the web camera. However, the accuracy and cost of the setup is a function of the frames per second (fps) catured by the web camera and both cost and accuracy increase together as the fps rating of a particular device incerases. The current setup (camera capturing at 18 fps) can provide satisfactory accuracy for a system oscillating at 1 Hz or below.

4.2.4 COMPARISON WITH THEORY:

For program refer Appendix I section A.

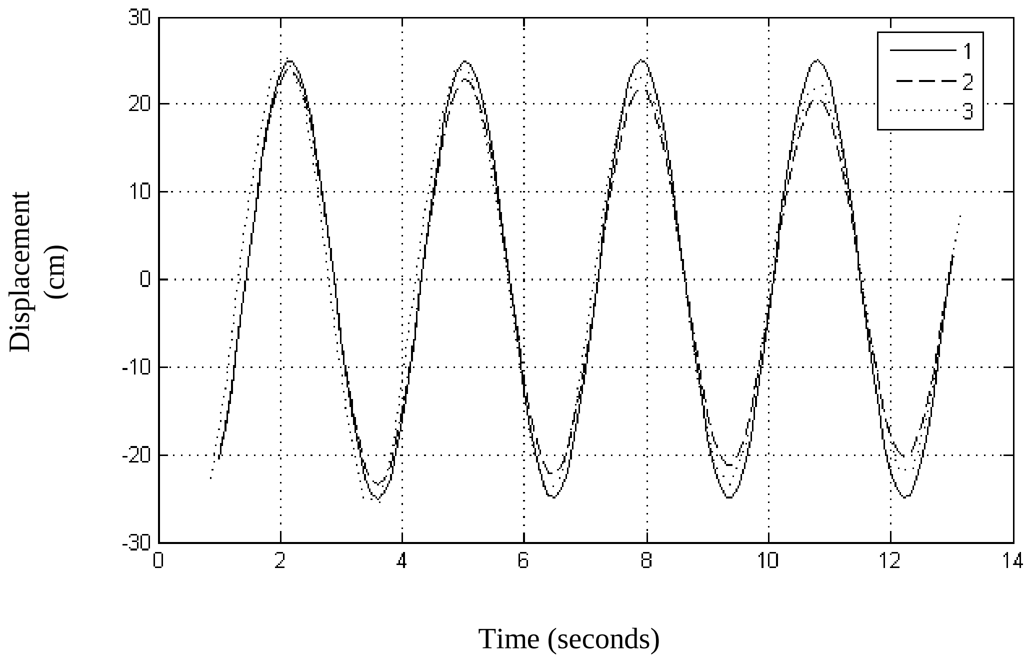


Figure 4.3: Comparison of Theoretical Undamped (1), Theoretical Damped (2) and Experimental Displacement (3)

4.2 THE DOUBLE PENDULUM

A double pendulum is a simple mechanical system which consists of one pendulum suspended from another. It is, thus, a two-degree-of-freedom system having two masses and each mass has a rotation about a pivot point.

A. Introduction

The first section of this project involves the development of an experimental setup to verify experimentally, the theoretical analysis of the motion of two-degree-of-freedom (2 DOF) systems. A two-degree-of-freedom system is one which requires two independent coordinates to describe its motion.

The study of vibrations of two-degree-of-freedom systems enables one to simplify complex systems having multiple degrees of freedom by modelling them to systems having two degrees of freedom (e.g. Vibrations of a Lathe machine, Pitching of Automobiles, Seismic vibrations in Structures, etc.). Analysis of these systems thus becomes possible by idealizing certain parameters to assume them as two-degree-of-freedom systems.

The number of degrees of freedom of a system may be given as:

Number of degrees of freedom of a system

= *Number of masses in the system*

×

Number of possible types of motion of each mass

A two-degree-of-freedom system is governed by two equations of motion, one for each degree of freedom. Each of these equations contain both the coordinates, and are called *coupled* equations, since they cannot be solved independently of each other. The basic two-degree-of-freedom system, the double pendulum, is selected to be developed for study under this section of the project. A good introduction to different two DOF systems can be found in [56].

A double pendulum is a simple mechanical system which consists of one pendulum suspended from another. It is, thus, a two-degree-of-freedom system having two masses and each mass has a rotation about a pivot point. An intuition would suggest a predictable motion, governed by simple laws of nature which could easily be studied and analyzed by easily solvable differential equations. However, contrary to this very intuition, the motion of the double pendulum is inherently chaotic in nature — a slight change in any of the initial conditions may result in a high degree of change in the motion of the pendulum. In chaotic systems, there is a close correlation between the system's motion and its initial conditions and the chaos evolves with each succeeding instant of time. These types of systems lead to complex nonlinear differential equations, for which analytical solutions do not exist. Such systems may only be solved numerically on a computer, since the efforts and time required to compute the necessary iterations would render it practically impossible to manually solve them.

The essence of studying the intriguing motion of this chaotic oscillator is to understand the underlying behavior of simple chaotic systems. Its deterministic chaotic behavior makes it a model system to investigate. This study may be extended to delve deeper into the theory of chaos and apply the findings to more complex applications such as electrical circuits, structural vibrations, weather patterns, fluid motion as well as computer hardware. It would somewhat increase our limited ability to predict the evolution of such systems over long periods of time. Even though Vibrations are present in almost all areas of science, it is perceived to be a difficult subject to understand. Demystifying the motion of the double pendulum would serve as a triumph over the inertia to study this fascinating subject.

4.2.1 EQUATIONS OF MOTION:

Consider the derivation of equations of motion of the compound pendulum as shown in Figure 4.4.

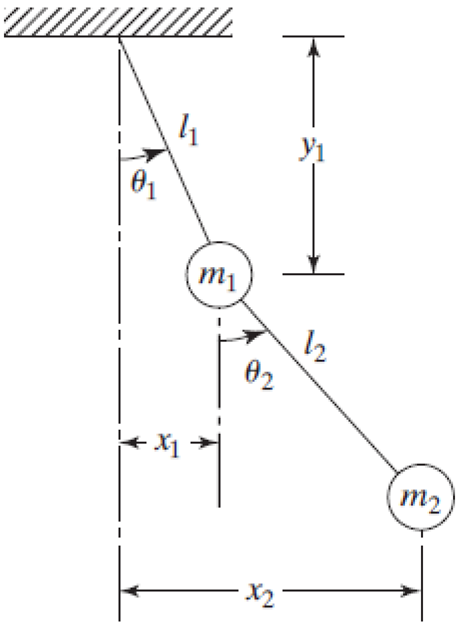


Figure 4.4: The Double Pendulum (Schematic)

Consider a double bob pendulum with masses m_1 and m_2 and attached by rigid, massless wires of lengths $l1$ and $l2$. Also, let the angles the wires make with the vertical be denoted as θ_1 and θ_2 Finally, let the acceleration due to gravity be g.

The positions of the bobs are given by the following equations:

$$x_1 = l_1 \sin(\theta_1) \tag{41}$$

$$y_1 = -l_1 \cos(\theta_2) \tag{42}$$

$$x_2 = l_1 \sin(\theta_1) + l_2 \sin(\theta_2) \tag{43}$$

$$y_2 = -l_1 \cos(\theta_1) - l_2 \cos(\theta_2) \tag{44}$$

The potential energy (V) of the system is then given by:

$$\mathrm{V} = m_1 g y_1 + m_2 g y_2 = -(m_1 + m_2) g l_1 \cos(\theta_1) - m_2 g l_2 \cos(\theta_2) \tag{45}$$

The kinetic energy (K) is given by:

$$K = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 = \frac{1}{2}m_1{l_1}^2{\dot{\theta}_1}^2 + \frac{1}{2}m_2[{l_1}^2{\dot{\theta}_1}^2 + {l_2}^2{\dot{\theta}_2}^2 + 2l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)] \quad (46)$$

The Lagrangian (L) is then:

$$\mathrm{L} \equiv T - V = \frac{1}{2}(m_1 + m_2){l_1}^2{\dot{\theta}_1}^2 + \frac{1}{2}m_2{l_2}^2{\dot{\theta}_2}^2 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + (m_1 + m_2)g l_1 \cos(\theta_1) + m_2 g l_2 \cos(\theta_2) \quad (47)$$

Now we can compute the generalized momenta, $p_i = \frac{\partial L}{\partial \dot{\theta}_i}$

$$p_{\theta_1} = \frac{\partial L}{\partial \dot{\theta}_1} = (m_1 + m_2){l_1}^2\dot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \quad (48)$$

$$p_{\theta_2} = \frac{\partial L}{\partial \dot{\theta}_2} = m_2{l_2}^2{\dot{\theta}_2}^2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_1 - \theta_2) \quad (49)$$

The Hamiltonian (H) is then given by:

$$H = \dot{\theta}_i p_i - L = \frac{1}{2}(m_1 + m_2){l_1}^2{\dot{\theta}_1}^2 + \frac{1}{2}m_2 l_2{\dot{\theta}_2}^2 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) - (m_1 + m_2)g l_1 \cos(\theta_1) - m_2 g l_2 \cos(\theta_2) \quad (50)$$

Solving the generalized momenta equations for q_1 and θ_2 and plugging back into the Hamiltonian equation:

$$H = \frac{m_2{l_2}^2{p_{\theta_1}}^2 + (m_1+m_2){l_1}^2{p_{\theta_2}}^2 - 2m_2 l_1 l_2 p_{\theta_1} p_{\theta_2} \cos(\theta_1 - \theta_2)}{2{l_1}^2{l_2}^2 m_2[m_1 + \sin^2(\theta_1 - \theta_2)m_2]} - (m_1 + m_2)g l_1 \cos(\theta_1) - m_2 g l_2 \cos(\theta_2) \quad (51)$$

This leads to the Hamilton’s Equations of Motion:

$$\dot{\theta}_1 = \frac{\partial H}{\partial p_{\theta_1}} = \frac{l_2 p_{\theta_1} - l_1 p_{\theta_2} \cos(\theta_1 - \theta_2)}{l^2{}_1 l_2(m_1 + m_2 sin^2(\theta_1 - \theta_2))} \quad (52)$$

$$\dot{\theta}_2 = \frac{\partial H}{\partial p_{\theta_2}} = \frac{l_1(m_1 + m_2)p_{\theta_2} - l_2 m_2 p_{\theta_2} \cos(\theta_1 - \theta_2)}{l^2{}_2 l_1(m_1 + m_2 sin^2(\theta_1 - \theta_2))} \quad (53)$$

$$\dot{p_{\theta_1}} = -\frac{\partial H}{\partial \theta_1} = -(m_1 + m_2)gl_1 \sin(\theta_1) - C_1 + C_2 \tag{54}$$

$$\dot{p_{\theta_2}} = -\frac{\partial H}{\partial \theta_2} = -m_2 g l_2 \sin(\theta_2) + C_1 - C_2 \tag{55}$$

$$C_1 = \frac{p_{\theta_1} p_{\theta_2} m_2 \, sin(\theta_1 - \theta_2)}{l_1 l_2 (m_1 + m_2 \, sin^2(\theta_1 - \theta_2)} \tag{56}$$

$$C_2 = \frac{[(l^2{}_1 p^2{}_{\theta_2}(m_1+m_2) + \left(l^2{}_2 p^2{}_{\theta_1} m_2\right) - (l_1 l_2 m_2 \, p_{\theta_1} p_{\theta_2} \cos(\theta_1 - \theta_2))] \sin(2(\theta_1 - \theta_2))}{2 l^2{}_1 l^2{}_2 (m_1 + m_2 \, sin^2(\theta_1 - \theta_2))^2} \tag{57}$$

4.2.2 CHAOS IN MECHANICAL SYSTEMS:

The double pendulum is what is called a Hamiltonian system, which essentially means that the total mechanical energy of the system is constant in time, or conserved. This is of course only approximately true over short period of time in the real world as there is some dissipation due to air resistance and internal friction in the bearings, Our system is also in principle deterministic, given exact initial conditions. Given these facts one might ask how it can undergo chaotic motion, and in what sense it may be considered chaotic.

These questions necessitate a brief discussion of what is meant by chaos. One definition is that chaotic systems are highly sensitive to initial conditions. In practice this means that if the initial conditions of the double pendulum (initial angles and angular velocities) are altered only slightly, the consequent evolution of the double pendulum will differ drastically.

This definition of chaos is somewhat qualitative, but it is very useful in practice and it encapsulates what is important about chaotic systems in physical applications, our limited ability to predict the evolution of a given system over long periods of time.

4.2.3 THE LYAPUNOV EXPONENT:

It is to be expected that the double pendulum behaves chaotically only in certain regimes, meaning that some sets of initial conditions lead to chaotic behavior, while others do not. It is expected that small initial angles (meaning that the pendula are displaced only slightly from the vertical) combined with small initial velocities will lead to non-chaotic behavior while larger initial angles and velocities lead to chaotic behavior. A system is said to evolve chaotically if an initial separation between two phase space trajectories grows exponentially with time. If initial separation vector in has is δx_0 and the separation at a time t is $\delta x(t)$ we may write

$$|\delta x(t)| = e^{\lambda t}|\delta x_0|$$

The system is said to behave chaotically if the Lyapunov exponent λ is positive, while it is less than or equal to zero denotes non-chaotic behavior. It is possible to define separate Lyapunov exponents for different directions in phase space, and the largest of these exponents is said to be the principal Lyapunov exponent. We calculate the principal Lyapunov in such a way that it does not correspond to one single direction in phase space.

4.2.4 THE EXPERIMENT:

The experiment was carried out by focusing the camera on the red and blue bobs of the Pendulum. Illumination LEDs were turned on to provide optimum tracing conditions. A video footage of the string was recorded. This footage was converted onto individual frames and processed with the code enlisted in Appendix I section B. Following is the algorithm for processing:

- Any Image can be designated as I_{rgb}
- Subtracting I_r from the image leaves with us an image called I_{gb}

$$I_{rgb} - I_r = I_{gb}$$

- Also, if apply the following transformation to the image matrix, we obtain

$$I_{rgb} - I_{bg} = I_r$$

- I_r after binarization and segmentation gives a binary image which can easily processed to obtain coordinate data of the object coded in color r.

It must be kept in mind that while color coding objects for recognition, only color shades that the computer designates as either Red, Blue or Green should be taken.

Following are some of the experimental conditions:

Length of Link 1 (Upper)	26 cm
Length of Link 2 (Lower)	36 cm
Mass of Bobs	330 grams
Initial Angle of Bob 1 (θ_1)	30°
Initial Angle of Bob 2 (θ_2)	0°

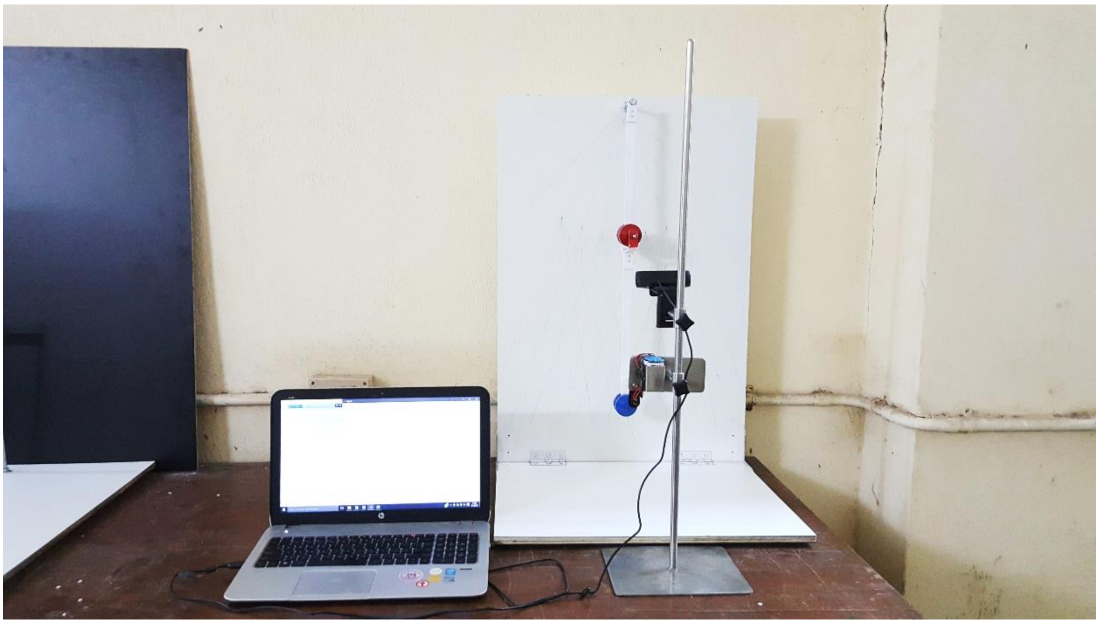


Figure 4.5: Experimental Set-up for the Study of Two DOF System

Figure 4.6: Sensor Stand with Illumination module

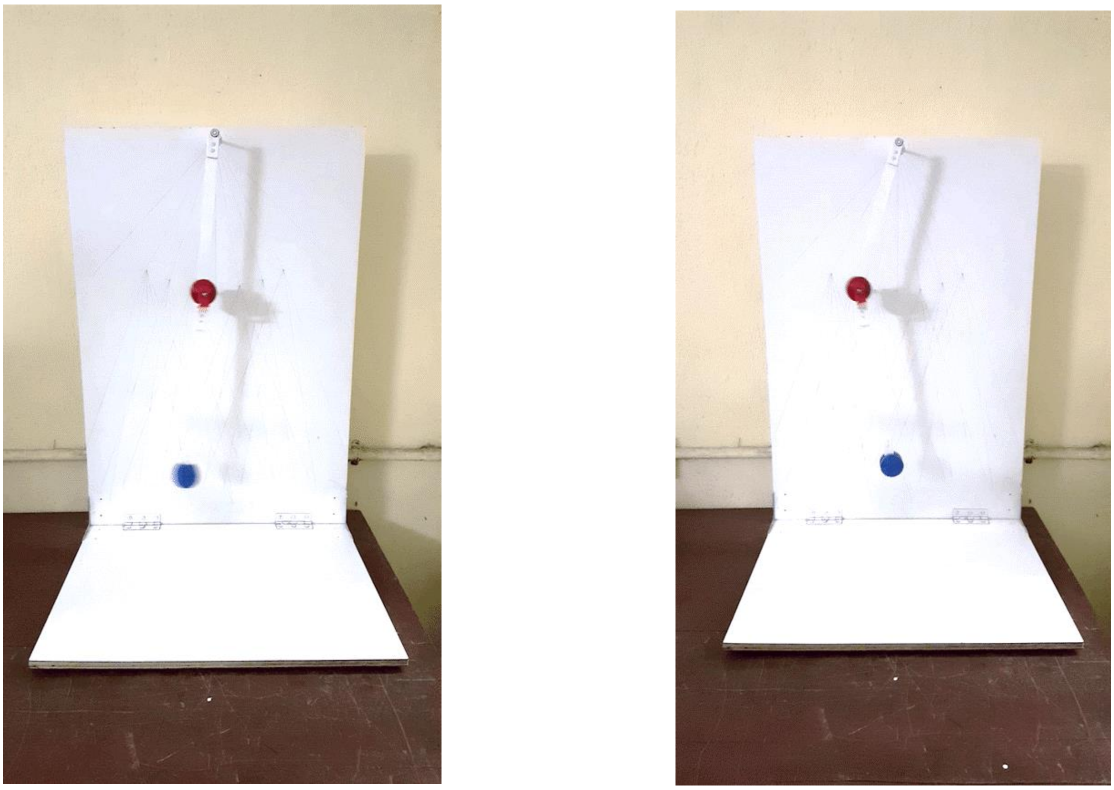


Figure 4.7: Mode 1 of Oscillation (Left), Mode 2 of Oscillation (Right) Double Pendulum

4.2.5 <u>EXPERIMENTAL RESULTS</u>:

Following graphs illustrate the experimental results obtained through video analysis of the data collected of the dual pendulum in Mode 1 (Some initial angle θ_1 and the links in a straight line, as against Mode 2 in which both θ_1 and θ_2 have different values)

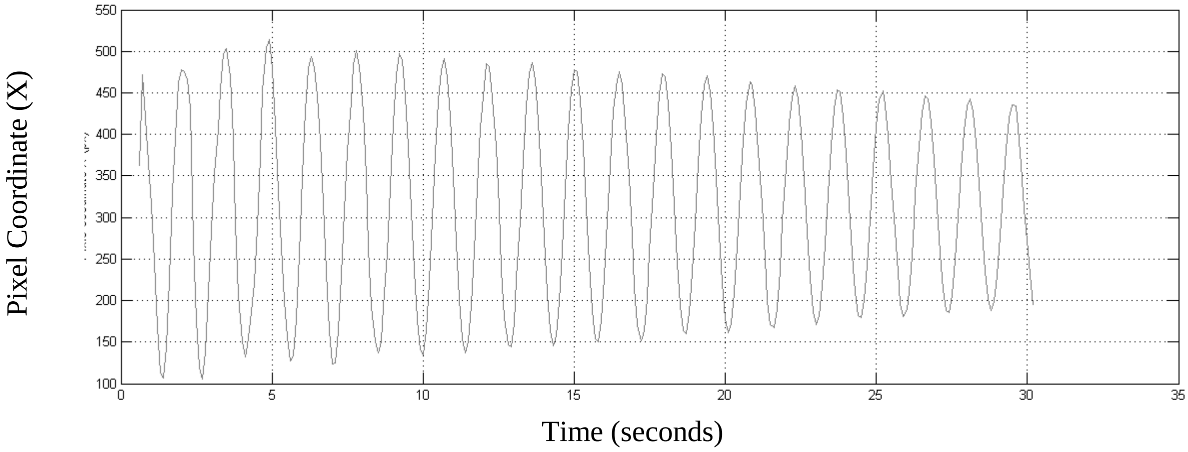


Figure 4.8: Pixel Coordinate (X) versus Time (Seconds) of Uncalibrated Motion of Bob1 in Mode 1

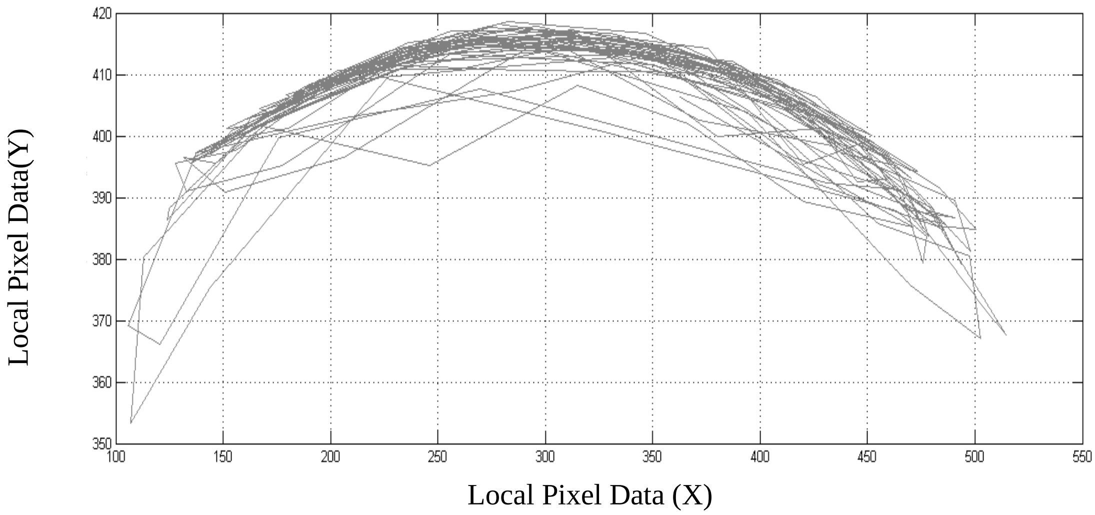


Figure 4.9: Motion of Bob 2 in Mode 1 (Uncalibrated, X Y Plot)

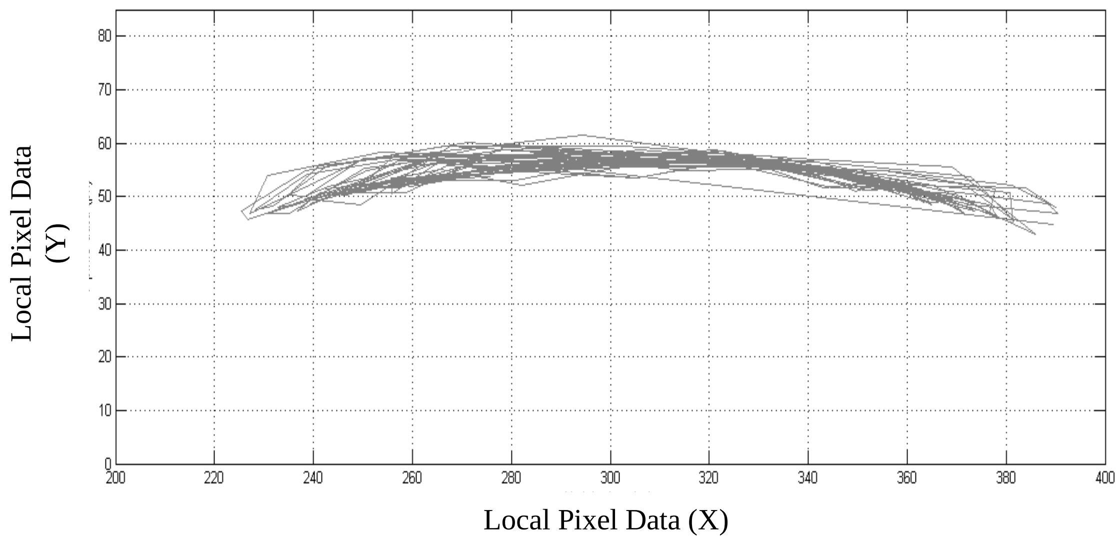


Figure 4.10: Motion of Bob 1 in Mode 1 (Uncalibrated, X Y Plot)

4.2.6 <u>COMPARISON WITH THEORY</u>:

Figure 4.11 contains the simulated double pendulum based on the Cartesian equation of motion of the double pendulum. The program for the below simulation is enlisted in Appendix I section D.

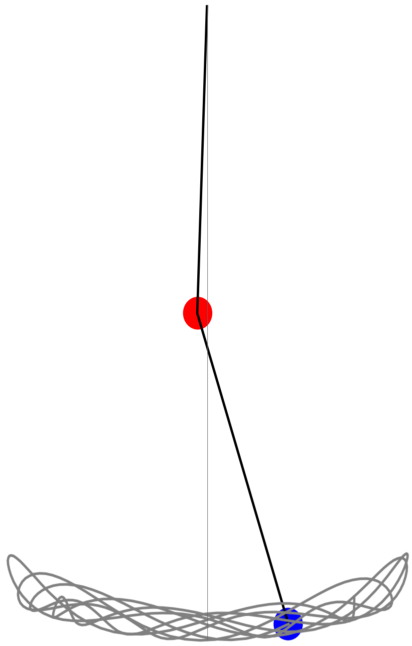


Figure 4.11: Simulation of the Double Pendulum in Mathematica

4.2.7 DISCUSSION:

Theoretical and practical aspects were studied. It was found that, as the double pendulum is highly sensitive to conditions such as initial angles and damping. It is difficult to obtain an exact correlation of theoretical and practical plots. The regimes of the plots obtained however were similar

A published paper on the simple pendulum approach is enlisted in the end of this report and can be found in [57].

CHAPTER 5: CONTINUOUS DOF SYSTEM

Continuous systems are those which have continuously distributed mass and Elasticity. These continuous systems are assumed to be homogeneous and isotropic obeying hook law within the elastic limit. Since to specify the position of every point in the continuous systems an infinite number of coordinates is required therefore a continuous systems is considered to have infinite number of degrees of freedom. Thus there will be infinite natural frequencies free vibration of continuous system is sum of the principal or normal modes. Continuous systems such as beams and strings have been studied since the 17^{th} century to model vibrations caused in bridges and cables.

5.1 INTRODUCTION

In this article we consider a string supported at both ends (tied to one end and kept under tension by a weight over a pulley at the other) as a continuous system.

5.2 THE SPECIFIC CONTINUOUS SYSTEM

Detailed motion analysis of a fixed-fixed type string is covered in the subsequent sub-chapters. Comparison of experimental data with theoretical solution is also done.

5.2.1 EQUATIONS OF MOTION:

In the continuous system, the equation of motion is derived from the free body diagram of an infinitesimally small element of the system with the help of the Newton's Second law of motion.

Here we are considering the string fixed at two end i.e. Fixed-Fixed type of string. The equation of motion for the string is derived as:

Consider an elastic string of length ***l***, subjected to the transverse force $f(x,t)$ per unit length. Transverse displacement is represented by $w(x,t)$ as shown in Fig.1. The net balancing forces acting on the system are given by:

$$(P + dp) * \sin(\theta + d\theta) + f dx - P * \sin\theta = \rho * dx \frac{\partial^2 w}{\partial t^2} \tag{58}$$

Where ρ is Mass per unit length and P is tension inside the string, θ is the angle of deflection of string with respect to x-axis.

As,

$$\sin(\theta + d\theta) \cong \tan(\theta + d\theta) = \frac{dw}{dx} + \frac{\partial^2 w}{\partial x^2} dx \tag{59}$$

the forced-vibration equation of the non-uniform string becomes,

$$P * \frac{\partial^2 w(x,t)}{\partial x^2} + f(x,t) = \rho * \frac{\partial^2 w(x,t)}{\partial t^2} \tag{60}$$

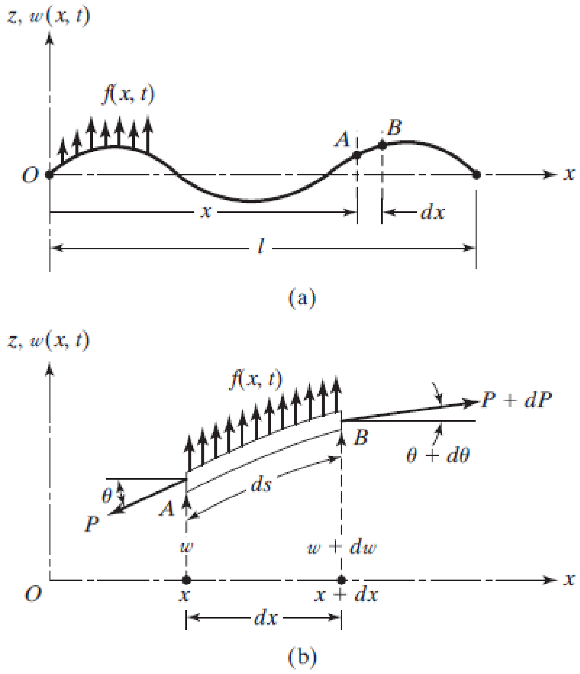


Figure 5.1: A Vibrating String

Considering f$(x,t) = 0$,

$$P * \frac{\partial^2 w(x,t)}{\partial x^2} = \rho * \frac{\partial^2 w(x,t)}{\partial t^2} \tag{61}$$

Or where,

$$c^2 * \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial t^2} \tag{62}$$

$$c = \sqrt{\frac{P}{\rho}} \tag{63}$$

This equation is known as wave equation or equation of motion for Fixed-Fixed string.

The free vibration equation can be solved by method of separation variable. In this method, transverse displacement $w(x,t)$ is the product of function $W(x)$ and $T(t)$.

$w(x,t) = W(x)T(t)$

For string fixed at both the ends, boundary conditions are,

$w(0,t) = w(l,t) = 0 \qquad \textit{for time } t \geq 0.$

For the above conditions the *n*th natural frequency of the system is given by,

$$\omega_n = \frac{nc\pi}{l}, \qquad n = 1,2,3, \ldots\ldots \tag{64}$$

Hence nth displacement corresponding to the ω_n is given by,

$$w_n(x,t) \;=\; W_n(x)T_n(t) = \sin\frac{n\pi x}{l}\left[\mathrm{Cn} * \cos\frac{nc\pi t}{l} + \mathrm{Dn} * \sin\frac{nc\pi t}{l}\right] \tag{65}$$

where Cn and Dn are arbitrary constants

The points at which ω_n=0, are called as the **nodes.** Thus fundamental has two nodes at x = 0 & x = l, 2nd node has three nodes, at x = 0, x = l/2 & x = l, etc. The mode shapes of string are shown in Figure 5.2.

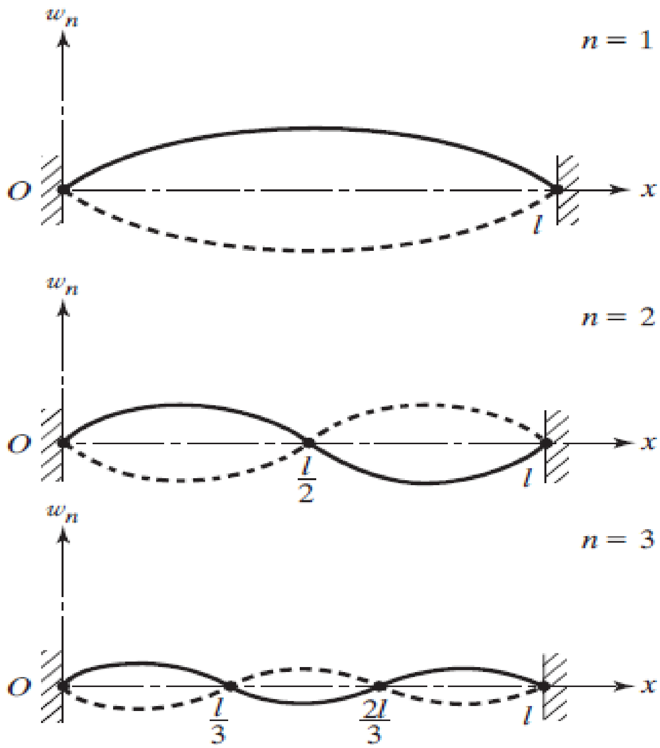


Figure 5.2: Mode Shapes of a String

The general solution for the wave equation is given by the superposition of all $w_n(x,t)$:

$$w(x,t) = \sum_{n=1}^{\infty} wn(x,t)$$

$$= \sum_{n=1}^{\infty} \sin\frac{n\pi x}{l}\left[\mathrm{C_n} * \cos\frac{nc\pi t}{l} + \mathrm{D_n} * \sin\frac{nc\pi t}{l}\right] \tag{66}$$

Values of Cn and Dn are find with the help of initial conditions:

$$\mathrm{C_n} = \frac{2}{1}\int_0^1 w_0(x) * \sin\frac{n\pi x}{l} * dx$$

$$D_n = \frac{2}{nc\pi}\int_0^1 \dot{w}_0(x) * \sin\frac{n\pi x}{l} * dx$$

This equation gives all possible vibrations of the string; the particular vibration that occurs is uniquely determined by the specified initial conditions. In this case we considered string plucked at mid-point as shown in Figure 5.3.

Since there is no initial velocity, $\dot{w}_0(x) = 0$, and so $D_n = 0$. Thus the equation of motion becomes,

$$w(x,t) = \sum_{n=1}^{\infty} C_n * \sin\frac{n\pi x}{l} * \cos\frac{nc\pi t}{l} \tag{67}$$

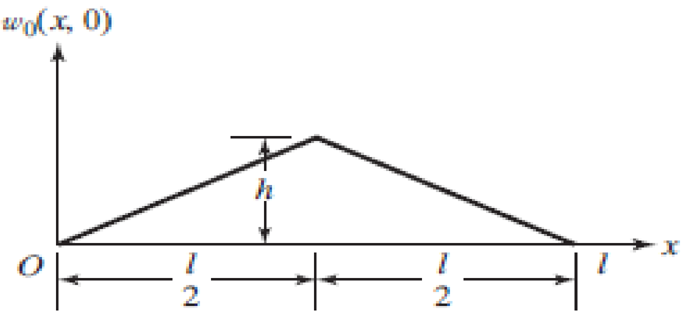


Figure 5.3: Initial Deflection of the String

Where,

$$Cn = \frac{2}{l}\int_0^1 w0(x) * \sin\frac{n\pi x}{l} * dx \tag{68}$$

The initial deflection $w_{0(x)}$ is given by,

$$w0(x) = \begin{cases} \frac{2hx}{l} & for\ 0 \le x \le \frac{l}{2} \\ \frac{2h(l-x)}{l} & for\ \frac{l}{2} \le x \le l \end{cases} \tag{69}$$

Substituting to get C_n :

$$Cn = \frac{2}{l}\left\{\int_0^{l/2} \frac{2hx}{l} * \sin\frac{n\pi x}{l} * dx + \int_{l/2}^{l} \frac{2h(l-x)}{l} * \sin\frac{n\pi x}{l} * dx\right\} \tag{70}$$

$$= \begin{cases} \frac{8h}{\pi^2 n^2} * \sin\frac{n\pi}{2} & for\ n = 1,3,5, \dots \dots \\ 0 & for\ n = 2,4,6, \dots \dots \end{cases}$$

Using this relation,

$$\sin\frac{n\pi}{2} = (-1)^{(n-1)/2} \qquad n = 1,3,5,\ldots \tag{71}$$

Hence displacement can be expressed as,

$$w(x,t) = \frac{8h}{\pi^2}\left\{\sin\frac{\pi x}{l} * \cos\frac{\pi ct}{l} - \frac{1}{9} * \sin\frac{3\pi x}{l}\cos\frac{3\pi ct}{l} + \cdots\right\} \tag{72}$$

In this case even harmonics are neglected.

5.2.2 THE EXPERIMENT:

The experiment was carried out by focusing the camera on the red marker on the string. Illumination LEDs were turned on to provide optimum tracing conditions. A video footage of the string was recorded and processed with the code enlisted in Appendix I section B.

Following are some of the experimental conditions:

Length of String	80 cm
Tension	0.981 N
Mass per Unit Length	0.00625 Kg/m

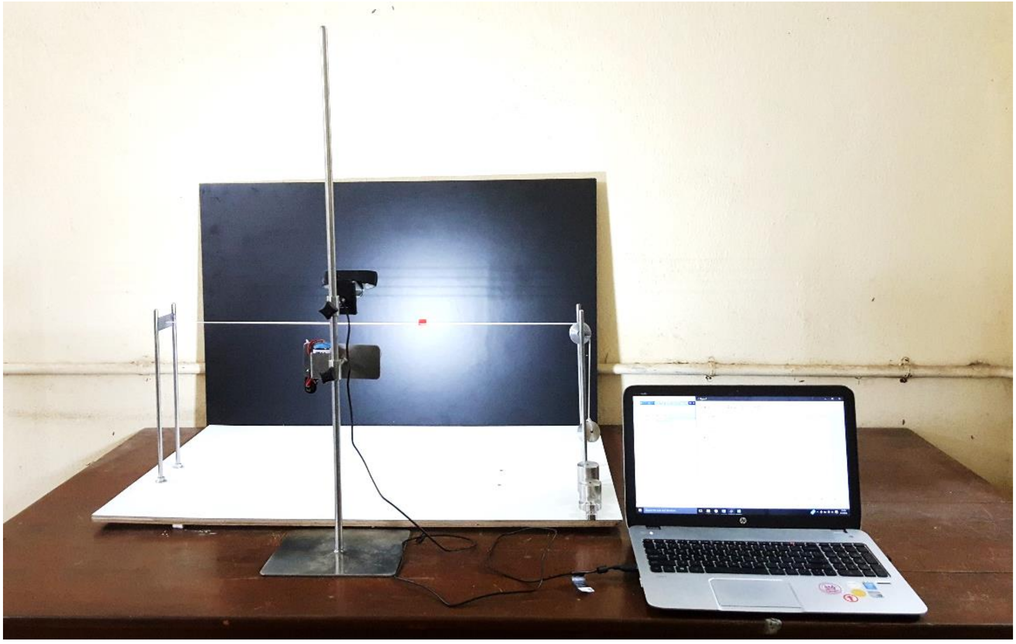


Figure 5.4: Experimental Set-up to Analyze the Vibration of a String

5.2.3 EXPERIMENTAL RESULTS:

The coordinate data of the red marker on the string was exported to an Excel® spreadsheet from MATLAB® where it was sorted to remove any garbage data. The plot in Figure 5.5 is a result of sorted garbage data, but still requires substantial amount of calibration and filtering.

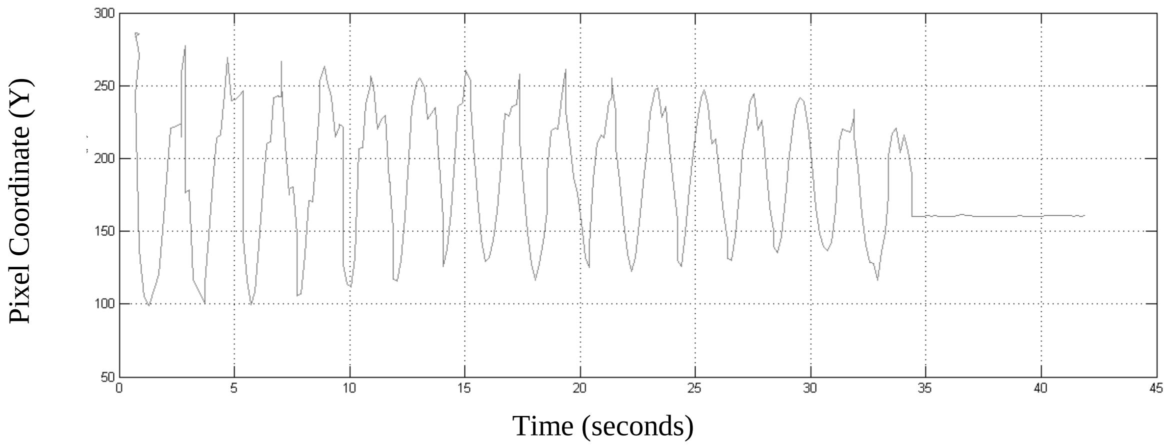


Figure 5.5: Experimental Results of a Vibrating String (Mode 1, Y Coordinate versus Time, Uncalibrated)

5.2.4 COMPARISON WITH THEORY:

Theoretical plot was obtained for the system with the code enlisted in Appendix I section C. The graph obtained is shown below in Figure 5.6.

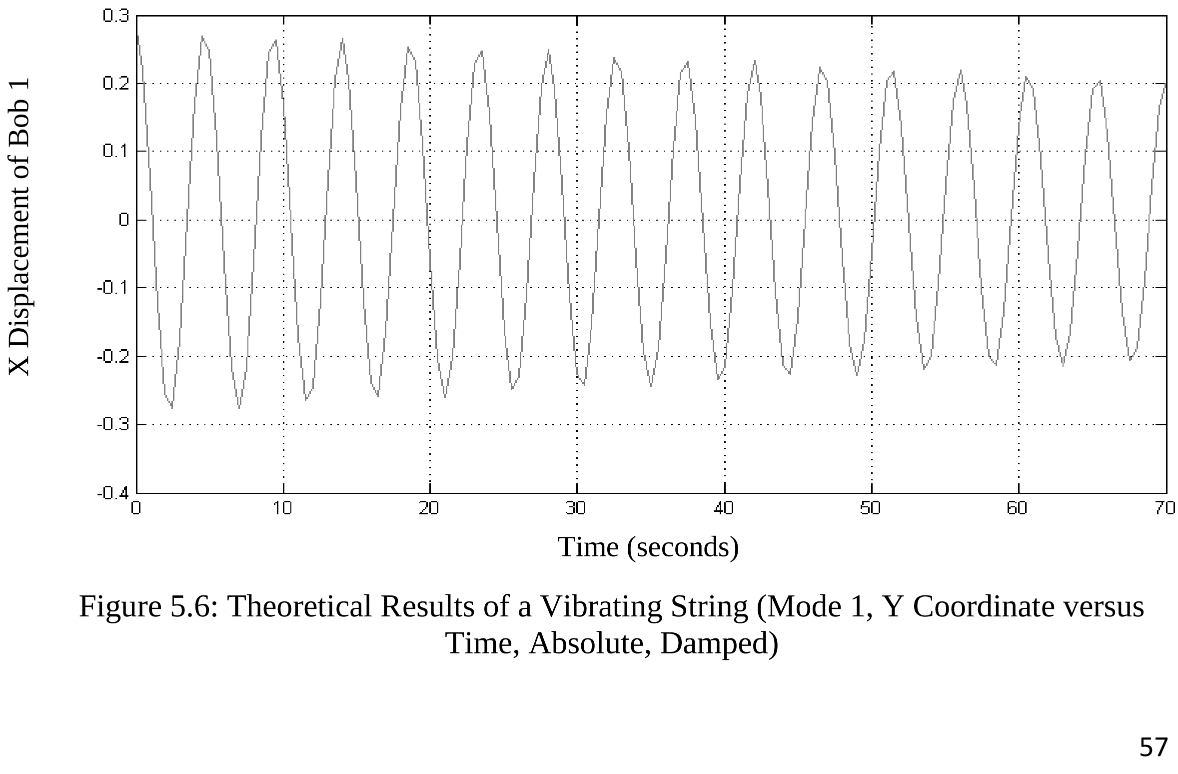


Figure 5.6: Theoretical Results of a Vibrating String (Mode 1, Y Coordinate versus Time, Absolute, Damped)

5.2.5 DISCUSSION:

Practical and theoretical results for a plucked string fixed at both ends were compared. It was observed that the nature of both the graphs was similar and matched when the experimental graph was multiplied with a suitable calibration factor.

CHAPTER 6: CONDITION MONITORING THROUGH VBVS

Condition Based Monitoring (CBM) implies that the vibrations of a particular component be continuously monitored and its vibration signatures for normal operation be stored, to be used as a reference against any changes or detection of difference in vibrations, indicative of the development of a fault. Such a system enables the users to detect and diagnose a fault well before it causes a catastrophic breakdown.

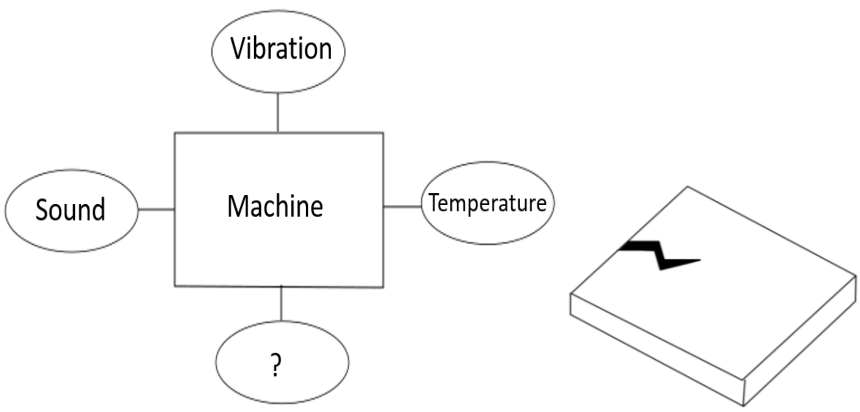


Figure 6.1: Fault Detection

The implications of the development of such Vibration Signature based Fault Detection Systems are spread through a variety of fields, from aerospace, to structures, robotics and many other forms of machinery. These systems contribute to reduction in maintenance costs by detecting faults and suggesting repairs of critical components at an appropriate time. Also, as they are many a times (but not always) non-intrusive, they do not affect the normal functioning of the machines.

After reviewing the available literature on Fault Detection using Vibration Analysis, we are reassured about the tremendous amount of applications of such a system. However, most of the equipment used was expensive.

Our objective under this particular project case is to develop a simple, cost effective, yet reliable model for fault detection for machine elements primarily through vision based signature analysis among others like sound, temperature et cetera. This system would be able to accurately sense the vibration signature of a particular component and detect irregularities or faults with acceptable precision. It would aid in monitoring the health of equipment and suggest the requirement of maintenance by proper diagnosis and thus be equivalent to a human keeping a close eye on a system.

CHAPTER 7: CONCLUSION AND FUTURE SCOPE

In the proposed method commonly available hardware and software tools were used to get a satisfactory degree of accuracy in extracting motion parameters of a damped simple pendulum, an ideal double pendulum using vision based image processing technique. The expenditure on setup indicate its viability as a low-cost solution implementable in laboratories of academic institutions especially those that are bound by funds. Further this approach may be extended to adapt for few more type of experimental studies covered in science and engineering curriculum like studying vibrations of multi degree of freedom and continuous degree of freedom systems.

Having said that, the existing hardware (the camera and processing platform) was found to be only fairly accurate for a continuous degree of freedom like a vibrating string, let alone a vibrating cantilever beam.

Future work of this project will constitute extending the methodology applied to double pendulum to suit some more oscillatory systems and expanding the existing 2D capability to 3D e.g. the conical pendulum. Applying the vision based technique to continuous system in a way that can yield better accuracy and reliability of the measured data. Also at present the set-up which started to be a portable, if not ultra-portable has only partially achieved its goal, Based on the surveyed literature the group believes in the feasibility of developing a multi-parameter condition monitoring device using vision based sensors to ‘look’ and detect faults.

APPENDIX I

A. MATLAB® program for analysis of simple pendulum:

```
t_1=[0.868782043
1.124691486
1.364778757
...dataset...

11.87672877
12.11678004
12.37270188
12.62874579
12.86868787
13.12471509
]
x=[-22.7061375
-11.0202
3.338133333
15.92525455
23.6798
25.42525455
...dataset...

-4.396515789
-15.25908889
-21.79327692
-21.60704211
-14.59135385
-3.685989474
7.703329412
];

m=mean(x);
x_1=x-m %x is pixel original data and x_1 is pixel data but sorted
x_2=(37.5/50)*x_1 %in cm 50px=37.5cm therefor 1px = 37.5/50    x_2 in in
global coordinates

%Theoretical  Undamped

%subplot(3,1,2)
t=1:0.1:13
```

```
w=-25*sin(2.1838*t)
plot(t,w,'g')
%grid on
%grid minor

%Theoretical  Damped
%subplot(3,1,3)

t=1:0.1:13
w_1=-25*exp(-0.0080*t*2.1838).*sin(sqrt(1-0.0080^2)*2.1838*t)
hold on
grid on
plot(t,w_1,'b')
%grid on
%grid minor
%hold off

% Observed Through Camera
%subplot(3,1,1)
%hold on
plot(t_1,x,'r')

%grid on
%grid minor
xlabel('Time(seconds)')
ylabel('Displacement in cm(x)')
hold off

[Maxima,MaxIdx] = findpeaks(x_2);

a_1 = log(Maxima(2)/Maxima(1))
zeta_1= (a_1/sqrt((2*pi)^2+a_1^2))

a_2 = log(Maxima(3)/Maxima(2))
zeta_2= (a_2/sqrt((2*pi)^2+a_2^2))

a_3 = log(Maxima(4)/Maxima(3))
zeta_3= (a_3/sqrt((2*pi)^2+a_3^2))

%hold on
%plot(t,0,'r')
%hold off
```

B. MATLAB® program for analysis of double pendulum consisting of blue bob:

```
%a = imaqhwinfo;
%[camera_name, camera_id, format] = getCameraInfo(a);
% code for BTP 2DOF
% Capture the video frames using the videoinput function
% You have to replace the resolution & your installed adaptor name.
vid = videoinput('winvideo',1);

% Set the properties of the video object
set(vid, 'FramesPerTrigger', Inf);
set(vid, 'ReturnedColorspace', 'rgb')
vid.FrameGrabInterval = 5;

% Crop the first frame.
%roi = [50 16 160 120];% cahnge according to your hardware
%firstFrame = f(:,:,:,1);
%frameRegion = imcrop(firstFrame, roi);

%start the videoaquisition here
start(vid)

% Set a loop that stop after 100 frames of aquisition
while(vid.FramesAcquired<=250)

    % Get the snapshot of the current frame
    datar = getsnapshot(vid);

    % Now to track red objects in real time
    % we have to subtract the red component
    % from the grayscale image to extract the red components in the image.
    diff_imr = imsubtract(datar(:,:,1), rgb2gray(datar));
    %Use a median filter to filter out noise
    diff_imr = medfilt2(diff_imr, [3 3]);
    % Convert the resulting grayscale image into a binary image.
    diff_imr = im2bw(diff_imr,0.18);
    % Remove all those pixels less than 300px
    diff_imr = bwareaopen(diff_imr,20);
    % Label all the connected components in the image.
    bwr = bwlabel(diff_imr, 8);

    % Here we do the image blob analysis.
```

```
    % We get a set of properties for each labeled region.
    statsr = regionprops(bwr, 'BoundingBox', 'Centroid');
    [f,t] = getdata(vid);
    % Display the image
    imshow(datar)

    hold on

    %This is a loop to bound the red objects in a rectangular box.
    for object = 1:length(statsr)
        bbr = statsr(object).BoundingBox;
        bcr = statsr(object).Centroid;
        rectangle('Position',bbr,'EdgeColor','r','LineWidth',2)
        dlmwrite('cont_final.xls', bcr, '-append')
        dlmwrite('t_cont_final.xls', t, '-append')
        %dlmwrite('f.xls', f, '-append')
        plot(bcr(1),bcr(2), '-m+')
        a=text(bcr(1)+15,bcr(2), strcat('X: ', num2str(round(bcr(1))), '
Y: ', num2str(round(bcr(2)))));
        set(a, 'FontName', 'Arial', 'FontWeight', 'bold', 'FontSize', 12,
'Color', 'yellow');
    end

    hold off

end
% Both the loops end here.

% Stop the video aquisition.
stop(vid);

% Flush all the image data stored in the memory buffer.
flushdata(vid);

% Clear all variables
clear all
```

C. MATLAB® program for theoretical plot of plucked string fixed at both ends:

```
%this is a MATLAB  plot for a plucked string fixed at both ends
t = 0:0.5:70

%c = (tension/mass per unit length)^1/2
c  = (0.060*9.81/(5/1))^0.5
%initial plucking height, refer diagram
h = 0.35
%length where the string is plucked
x = 0.40
%total length of the string
l = 0.80
%pi constant
pi = 3.14

%Equation governing the dynamics for modes, n = 1, 3, 5, .....
y = (8*h/pi^2)*(sin(pi*x/l)*cos(pi*c*t/l)-
(1/9)*sin(3*pi*x/l)*cos(3*pi*c*x/l))
%For damped response add .*exp(-0.0045*t) at end
% 0.0045 is unscaled approximate damping coefficient


%Plotting the ycoordinate of string versus time
plot(t,y)
grid on
```

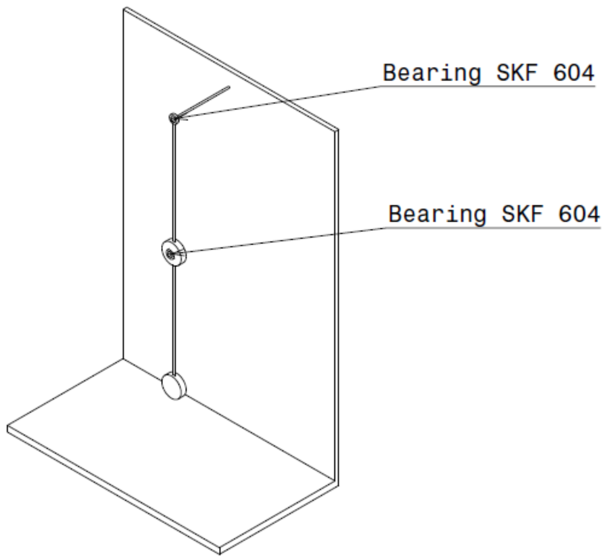
D. Mathematica® program for theoretical simulation of double pendulum:

```
deqns={Subscript[m,1] x1''[t]==(λ1[t]/Subscript[l,1]) x1[t]-
(λ2[t]/Subscript[l,2]) (x2[t]-x1[t]),Subscript[m,1]
y1''[t]==(λ1[t]/Subscript[l,1]) y1[t]-(λ2[t]/Subscript[l,2]) (y2[t]-y1[t])-
Subscript[m,1] g,Subscript[m,2] x2''[t]==(λ2[t]/Subscript[l,2]) (x2[t]-
x1[t]),Subscript[m,2] y2''[t]==(λ2[t]/Subscript[l,2]) (y2[t]-y1[t])-
Subscript[m,2] g};


aeqns={x1[t]^2+y1[t]^2==Subscript[l,1]^2,(x2[t]-x1[t])^2+(y2[t]-
y1[t])^2==Subscript[l,2]^2};


ics={x1[0]==0.5,y1[0]==-0.866025,y1'[0]==0,x2[0]==0.5,y2[0]==-
1.866025,y2'[0]==0};

params={g->9.81,Subscript[m,1]->1,Subscript[m,2]->1,Subscript[l,1]-
>1,Subscript[l,2]->1};


soldp=First[NDSolve[{deqns,aeqns,ics}/.params,{x1,y1,x2,y2,λ1,λ2},{t,0,15},
Method->{"IndexReduction"->{"Pantelides","ConstraintMethod"-
>"Projection"}}]];


Animate[Graphics[{{PointSize[.025],{Red,Point[{x1[t],y1[t]}]},{Blue,Point[{
x2[t],y2[t]}]},Line[{{0,0},{x1[t],y1[t]},{x2[t],y2[t]}}]}/.soldp,{Gray,Line
[Map[Function[Evaluate[{x2[#],y2[#]}/.soldp]],Range[0,t,0.025]]]}},PlotRang
e->{{-2,2},{-2.5,0}},Axes->True,Ticks->False,ImageSize-
>600],{t,0,10,.025},SaveDefinitions->True]
```

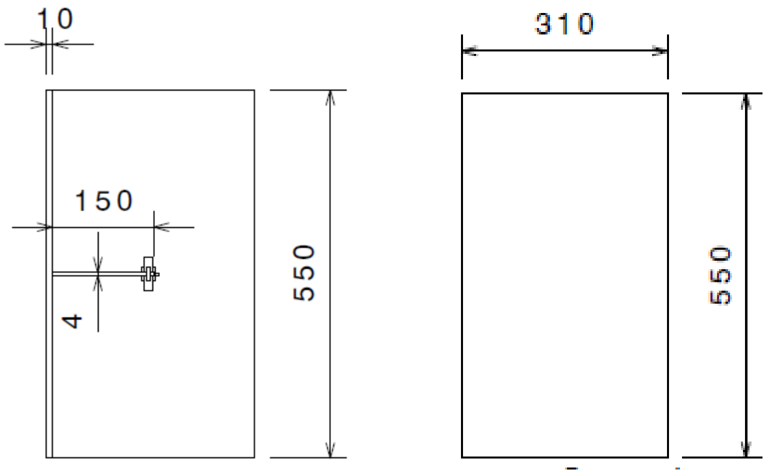
APPENDIX II

A. Drafts of one and two degree of freedom modular setup:

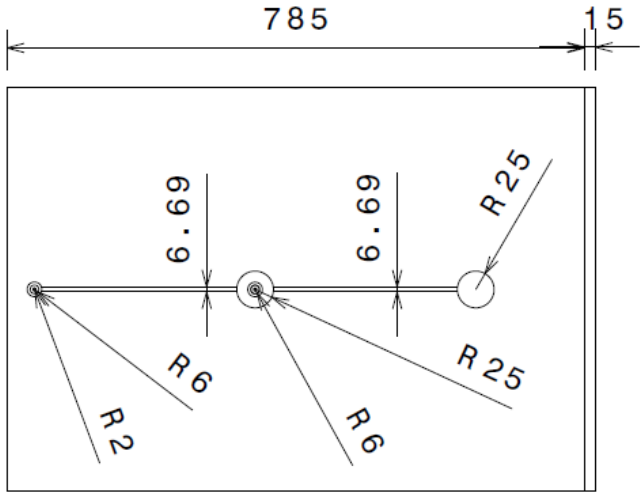
All dimensions in millimeter



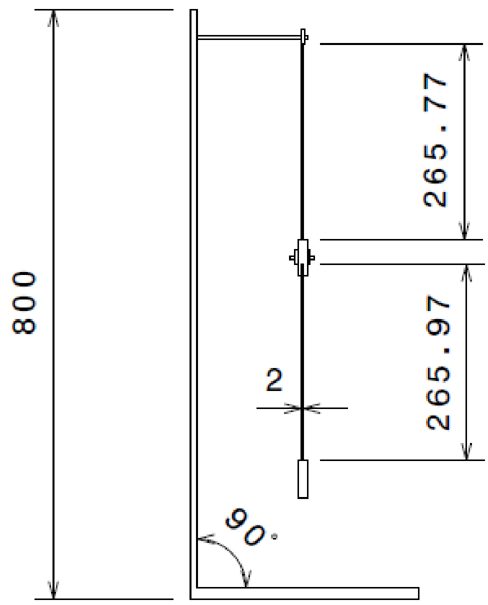
Isometric View



Top View (left), Bottom View (right)



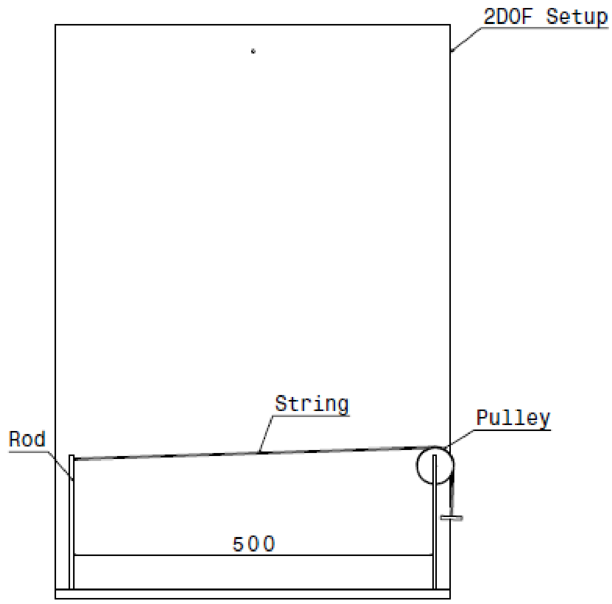
Front View



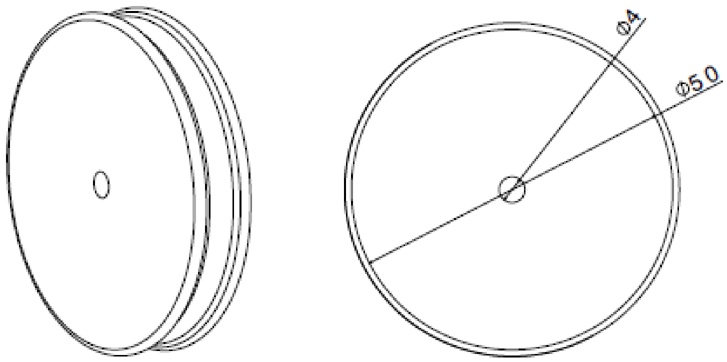
Side View

B. Drafts of continuous degree of freedom modular setup:

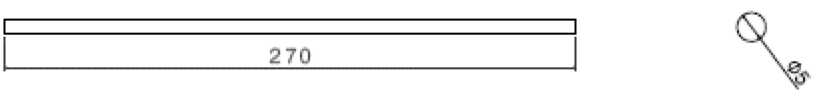
All dimensions in millimeter



Front View
(support board same as one and two DOF system setup)



Pulleys



Support Rod for Pulley and String

PUBLICATION

S. Meshram and N. Raykar, “Vision Based Approach to Experimental Study of Vibrations in Academia,” *Int. J. Adv. Eng. Sci. Technol.*, pp. 374–378, 2015.

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REFERENCES

[1] S. S. Rao, *Mechanical Vibrations*, 5th ed. University of Miami: Prentice Hall, 2010.

[2] A. Belendez, C. Pascual, D. I. Mendez, T. Belendez, and C. Neipp, “Exact solution for the nonlinear pendulum,” *Rev. Bras. Ensino Fis.*, vol. 29, no. 4, pp. 645–648, 2007.

[3] M. I. Qureshi and K. A. Quraishi, “Analytical Solution of Differential Equation Associated with Simple Pendulum,” vol. 3, no. 2, pp. 50–58, 2011.

[4] T. Shibata, “Precise Asymptotics of Boundary Layers for Damped Simple Pendulum Equations,” *Results Math.*, vol. 58, no. 1–2, pp. 105–118, Jan. 2010.

[5] M. Suzuki and I. S. Suzuki, “Physics of simple pendulum,” pp. 1–60, 2008.

[6] R. Nelson and M. Olsson, “The pendulum-Rich physics from a simple system,” 1985. [Online]. Available: http://fy.chalmers.se/~f7xiz/TIF081C/pendulum.pdf. [Accessed: 13-Oct-2015].

[7] E. K. Dunn, “The Effect of String Drag on a Pendulum,” no. 4, pp. 1–4, 2012.

[8] A. Arora, R. Rawat, S. Kaur, and P. Arun, “Study of the Damped Pendulum,” pp. 1–19, 2006.

[9] M. Abdelkader, “A damped simple pendulum of constant amplitude,” *Arch. Ration. Mech. Anal.*, vol. 85, no. 1, 1984.

[10] C. Medina, “Experimental Control of Simple Pendulum Model,” *Sci. Educ.*, vol. 13, no. 7–8, pp. 631–640, Nov. 2004.

[11] M. Fowler, “Using Excel to Simulate Pendulum Motion and Maybe Understand Calculus a Little Better,” *Sci. Educ.*, vol. 13, no. 7–8, pp. 791–796, Nov. 2004.

[12] V. Gintautas and A. Hubler, “A simple, low-cost, data-logging pendulum built from a computer mouse,” *Phys. Educ.*, vol. 44, no. 5, p. 3, 2009.

[13] D. M. Auslander, “Animating Physics,” Berkeley, 1999.

[14] “Calculating the Acceleration of Gravity Using an Accelerometer Data Logger,” *Gulf Coast Data Concepts*, 2016. [Online]. Available: http://www.gcdataconcepts.com/pendulum.html. [Accessed: 01-Jun-2016].

[15] R. Kavithaa, U. B. R, and D. Cr, “Simple Pendulum analysis – A Vision based approach,” no. Figure 1, 2013.

[16] B. A. Usman and M. S. Anwar, “Analyzing Simple Pendulum Phenomenon,” pp. 4–8, 2014.

[17] W.-K. Wong, T.-K. Chao, P.-R. Chen, Y.-W. Lien, and C.-J. Wu, “Pendulum experiments with three modern electronic devices and a modeling tool,” *J. Comput.*

Educ., vol. 2, pp. 77–92, 2015.

[18] "Modeling of a Simple Pendulum," 2016. [Online]. Available: http://ctms.engin.umich.edu/CTMS/index.php?aux=Activities_Pendulum. [Accessed: 25-May-2016].

[19] "The Simple Pendulum." [Online]. Available: http://edge.rit.edu/edge/P12361/public/Simple Pendulum. [Accessed: 25-May-2016].

[20] "The Simple Pendulum," 2005. [Online]. Available: http://nano-optics.colorado.edu/fileadmin/Teaching/phys1140/lab_manuals/LabManualM1.pdf. [Accessed: 22-Nov-2015].

[21] Q. Zhou, Ed., *Theoretical and Mathematical Foundations of Computer Science*, vol. 164. Berlin, Heidelberg: Springer Berlin Heidelberg, 2011.

[22] "Simple Pendulum Measurements With Hall Probe," 2014. [Online]. Available: http://qmplus.qmul.ac.uk/pluginfile.php/337311/mod_label/intro/simple_pendulum.pdf . [Accessed: 25-Jan-2016].

[23] "Calculating the Length of a Pendulum in Motion - MATLAB & Simulink Example - MathWorks India." [Online]. Available: http://in.mathworks.com/help/imaq/examples/calculating-the-length-of-a-pendulum-in-motion.html. [Accessed: 18-Jun-2015].

[24] "Dynamic Analysis of a Simple Pendulum Simulink Model for the Simple Pendulum," 2001. [Online]. Available: https://engineering.purdue.edu/~andrisan/Courses/AAE421_S2001/Docs_Out/Pendulum/PendulumHandOut.pdf.

[25] M. Z. Rafat, M. S. Wheatland, and T. R. Bedding, "Dynamics of a double pendulum with distributed mass," *Am. J. Phys.*, vol. 77, no. 3, p. 20, 2008.

[26] S. Sen, "A Double Pendulum : Fringing into Non-Linearity," *Beats Nat. Sci.*, vol. 1, no. 1, pp. 1–12, 2014.

[27] R. Nunna and A. Barnett, "Numerical Analysis of the Dynamics of a Double Pendulum," 2009.

[28] W. Hai, X. Liu, J. Fang, X. Zhang, W. Huang, and G. Chong, "Analytically bounded and numerically unbounded compound pendulum chaos," no. October, pp. 54–59, 2000.

[29] K. Vogt, "Hamiltonian Chaos : The Double Pendulum The physical system," 2006.

[30] D. Hoskin, "The Motion of a Pendulum," 2004. [Online]. Available: http://www.mubblemaths.co.uk/uploads/BSc.pdf.

[31] D. Arnab and T. Riziman, "Double Pendulum," 2010.

[32] D. G. Kelly, "Design, Fabrication, and Testing of a Driven Double Pendulum," James

Madison University, 2008.

[33] O. K. Ukoba, B. A. Olunlade, and W. Shelby, “Model-Based Object Tracking of Moving Object : Double Pendulum .,” *Pacific J. Sci. Technol.*, vol. 12, no. 1, pp. 170–176, 2011.

[34] R. Nielsen, F. Have, and B. Nielsen, “The Double Pendulum,” vol. 2, 2013.

[35] N. Kiritsis, Y. Huang, and D. Ayrapetyan, “A multi-purpose vibration experiment using Labview,” *Am. Soc. Eng. Educ. Annu. Conf. Expo. Copyr.*, 2003.

[36] “Strain Gauge Dynamic Testing,” 2010. [Online]. Available: http://web.cecs.pdx.edu/~sailor/CoursePages/ME411_Win10/Expt_2_BeamFrequency.pdf.

[37] A. Stanbridge and D. Ewins, “Modal testing using a scanning laser Doppler vibrometer,” *Mech. Syst. Signal Process.*, vol. 13, no. 2, pp. 255–270, 1999.

[38] M. Romaszko, B. Sapiński, and A. Sioma, “Forced vibrations analysis of a cantilever beam using the vision method,” *J. Theor. Appl. Mech.*, p. 243, 2015.

[39] B. Peeters, K. Peeters, H. Van der Auweraer, T. Olbrechts, F. Demeester, and L. Wens, “<title>Experimental modal analysis using camera displacement measurements: a feasibility study</title>,” in *Sixth International Conference on Vibration Measurements by Laser Techniques: Advances and Applications*, 2004, pp. 298–309.

[40] B. R. Jooste, H. J. Viljoen, S. L. Rohde, and N. F. J. Van Rensburg, “Experimental and theoretical study of vibrations of a cantilevered beam using a ZnO piezoelectric sensor,” *Am. Vac. Soc.*, pp. 714–719, 1996.

[41] J. Snamina, “INVESTIGATION ON VIBRATIONS OF A CANTILEVER BEAM WITH MAGNETORHEOLOGICAL FLUID BY USING THE ACOUSTIC SIGNAL,” *Act. Noise Vib. Control Methods*.

[42] H. Van der Auweraer, H. Steinbichler, C. Haberstok, R. Freymann, and D. Storer, “Integration of pulsed-laser ESPI with spatial domain modal analysis: results from the salome project,” *Proc. 4th Int. Conf. Vib. Meas. by Laser Tech. Adv. Appl.*, vol. 4072, pp. 29–42, 2000.

[43] F. Chen, W. . Luo, M. Dale, A. Petniunas, P. Harwood, and G. . Brown, “High-speed ESPI and related techniques: overview and its application in the automotive industry,” *Opt. Lasers Eng.*, vol. 40, no. 5–6, pp. 459–485, Nov. 2003.

[44] Y. Moheadin, “Using analytical and numerical approaches for crack monitoring based on the fundamental frequency of cracked beams,” *Int. J. Sci. Technol.*, vol. 26, no. 3, pp. 1824–1831, 2014.

[45] a. W. Smyth, S. F. Masri, T. K. Caughey, and N. F. Hunter, “Surveillance of Mechanical Systems on the Basis of Vibration Signature Analysis,” *J. Appl. Mech.*, vol. 67, no. 3, p.

540, 2000.

[46] C. Kar and A. R. Mohanty, “Vibration and current transient monitoring for gearbox fault detection using multiresolution Fourier transform,” *J. Sound Vib.*, vol. 311, no. 1–2, pp. 109–132, Mar. 2008.

[47] M. Runde, G. E. Ottesen, B. Skyberg, and M. Ohlen, “Vibration analysis for diagnostic testing of circuit-breakers,” *IEEE Trans. Power Deliv.*, vol. 11, no. 4, pp. 1816–1823, 1996.

[48] Y. Qu, E. Bechhoefer, D. He, and J. Zhu, “A New Acoustic Emission Sensor Based Gear Fault Detection Approach,” *Int. J. Progn. Heal. Manag.*, vol. 4, pp. 1–14, 2013.

[49] M. Morbidini, “A COMPARISON OF THE VIBRO-MODULATION AND THERMOSONIC NDT TECHNIQUES,” University of London, 2007.

[50] H. P. Kaumudi and V. Natarajan, “The simple pendulum: Not so simple after all!,” *Resonance*, vol. 14, no. 4, pp. 357–366, Aug. 2009.

[51] M. R. Matthews, *Time for Science Education*, vol. 8. Dordrecht: Springer Netherlands, 2000.

[52] P. E. Ariotti, “Aspects of the conception and development of the pendulum in the 17th century,” *Arch. Hist. Exact Sci.*, vol. 8, no. 5, pp. 329–410, 1972.

[53] C. Gauld, “The Treatment of the Motion of a Simple Pendulum in some Early 18th Century Newtonian Textbooks,” *Sci. Educ.*, vol. 13, no. 4/5, pp. 321–332, Jul. 2004.

[54] L. P. Pook, *Understanding Pendulums*, vol. 12. Dordrecht: Springer Netherlands, 2011.

[55] Y. Miao, F. Gao, and D. Pan, “Compound Pendulum Modeling and Resonant Frequency Analysis of the Lower Limbs for the Wearer and Exoskeleton,” *J. Bionic Eng.*, vol. 12, no. 3, pp. 372–381, 2015.

[56] “Two Degree of Freedom Systems.” [Online]. Available: http://www.iitg.ernet.in/scifac/qip/public_html/cd_cell/chapters/r_tiwari_dyn_of_mach/Chapter_12_Vibration of two-degree-of-freedom system.pdf. [Accessed: 25-Mar-2016].

[57] S. Meshram and N. Raykar, “Vision Based Approach to Experimental Study of Vibrations in Academia,” *Int. J. Adv. Eng. Sci. Technol.*, pp. 374–378, 2015.